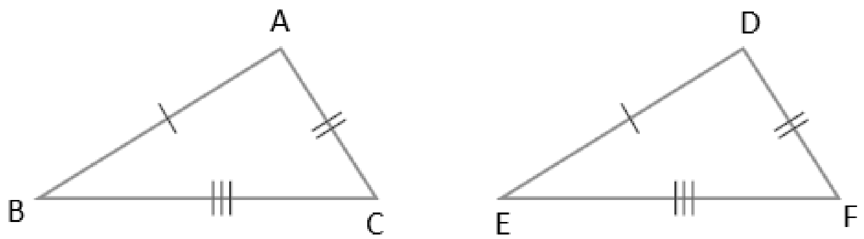
The SAS, ASA and SSS Postulates

To prove that two triangles are congruent we have to prove that three corresponding parts are congruent. Those corresponding parts can be sides only or a combination of sides and angles.

The postulates are used to prove that two triangles are congruent

Postulate 1: Side-Side-Side Congruency Postulate: (SSS Postulate)

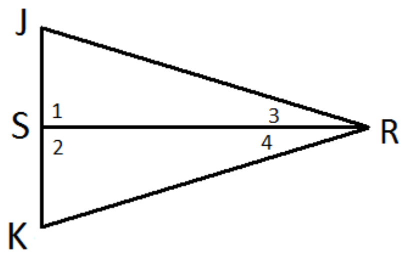
If three sides of one triangle equal the corresponding parts of the other, then the triangles are congruent.



$$\text{If }\left.\begin{array}{l}\overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{AC} \cong \overline{DF}\end{array}\right\} \Rightarrow \triangle BAC \cong \triangle EDF$$

Example 1:

Given: $\overline{RS}$ bisects $\overline{JK}$ $\overline{RJ} \cong \overline{RK}$

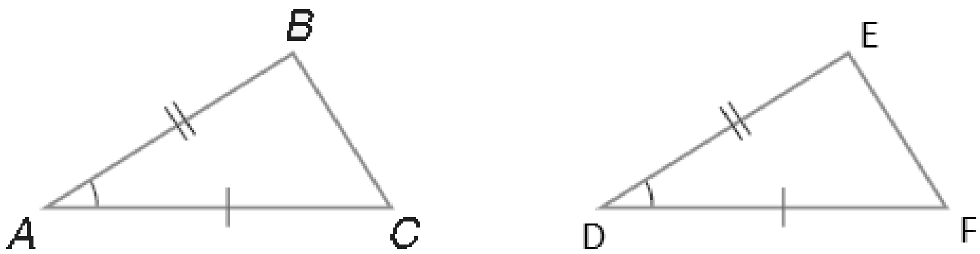


Prove: ΔRSJ $\cong$ ΔRSK

Statements	Reasons
1) $\overline{RS}$ **bisects** $\overline{JK}$	1) Given
2) $\overline{JS} \cong \overline{KS}$	2) Definition of a segment bisector
3) $\overline{RS} \cong \overline{RS}$	3) Reflexive
4) **ΔRSJ ≅ ΔRSK**	4) SSS postulate

Postulate 2: Side-Angle-Side Congruency Postulate: (SAS Postulate)

If two sides and the included angle of one triangle equal the corresponding parts of the other, then the triangles are congruent



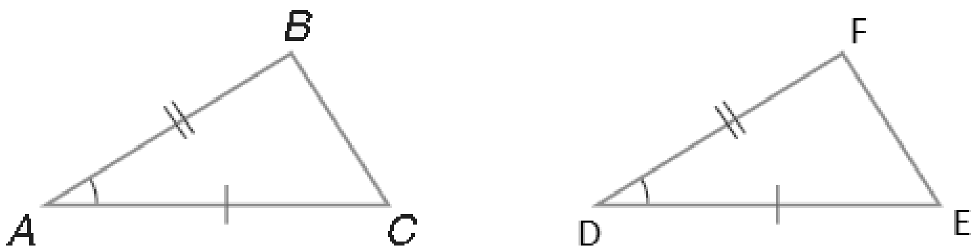
$$\text{If } \left.\begin{array}{l} \overline{AB} \cong \overline{DE} \\ \angle BAC \cong \angle EDF \\ \overline{AC} \cong \overline{DF} \end{array}\right\} \Rightarrow \triangle BAC \cong \triangle EDF$$

Example 2:

Given: $\overline{AB} \cong \overline{DF}$

$\overline{AC} \cong \overline{DE}$

$m\angle EDF = m\angle CAB$

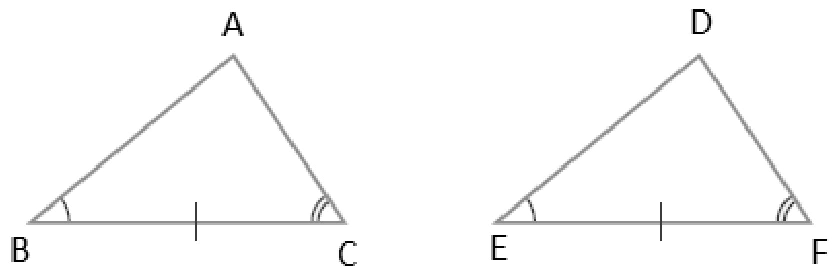


Prove: $\triangle EDF \cong \triangle CAB$

Statements	Reasons
1) $\overline{AB} \cong \overline{DF}$	1) Given
2) $m\angle EDF \cong m\angle CAB$	2) Given
3) $\overline{AC} \cong \overline{DE}$	3) Given
4) $\therefore\ \triangle EDF \cong \triangle CAB$	4) SAS Postulate

Postulate 3: Angle-Side-Angle Congruency Postulate: (ASA Postulate)

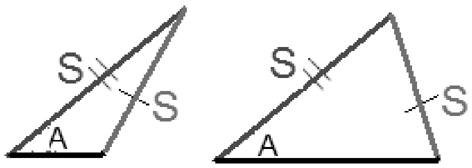
If two angles and the included side of one triangle equal the corresponding parts of the other, then the triangles are congruent



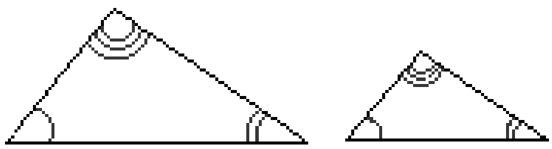
$$\text{If } \left.\begin{array}{l} \angle ABC \cong \angle DEF \\ \overline{BC} \cong \overline{EF} \\ \angle ACB \cong \angle DFE \end{array}\right\} \Rightarrow \triangle BAC \cong \triangle EDF$$

Two cases will not necessarily give congruent triangles are SSA and AAA so do not think about them when you want to prove congruent triangles

<u>SSA</u>**: Two triangles with two sides and a non-included angle equal may or may not be congruent.**

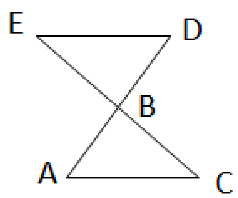


<u>**AAA:**</u> **If two angles on one triangle are equal, respectively, to two angles on another triangle, then the triangles are not necessarily congruent.**



<u>**Example 3:**</u>

Given: B is the midpoint of $\overline{AD}$ and $\overline{CE}$



Prove: $\triangle ABC \cong \triangle DBE$

Statements	Reasons
1) **B is the midpoint of $\overline{AD}$ and $\overline{CE}$**	1) Given
2) $\overline{EB} \cong \overline{CB}; \overline{AB} \cong \overline{DB}$	2) Definition of a midpoint
3) $\angle EBD \cong \angle CBA$	3) Vertical angles are congruent
4) **$\triangle ABC \cong \triangle DBE$**	4) SAS postulate