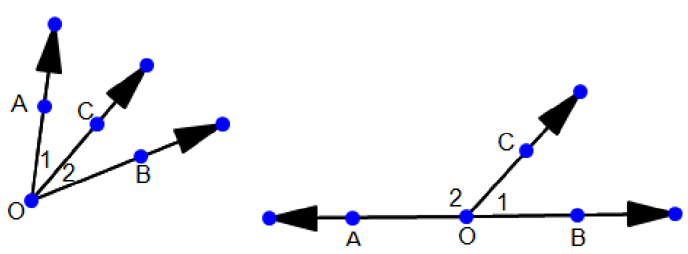
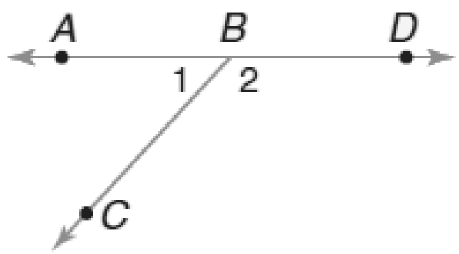
Supplementary and Complementary Angles



Each figure above shows a pair of adjacent angles, ∠1 and ∠2. In the second figure, the outer rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ form a line. In this case, ∠AOB is a straight angle and the outer rays are opposite rays.

Definition 1: Linear Pair

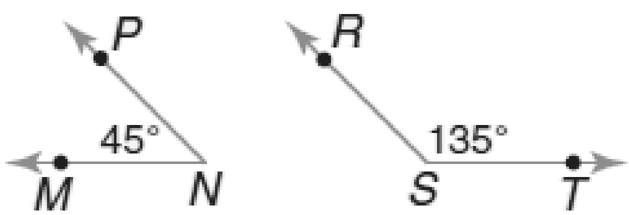
Two angles form a linear pair if and only if they are adjacent and their non-common sides are opposite rays



$\angle 1$ and $\angle 2$ are a linear pair

Definition 2: Supplementary Angles

Two angles are supplementary if and only if the sum of their degree measures is 180°

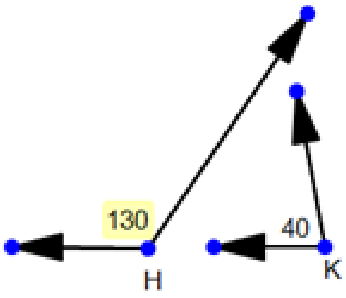


$m\angle MNP + m\angle RST = 180^{\circ}$

Each angle is called a supplement of the other.

Example 1: Are the pairs of angles supplementary? Explain your answer.

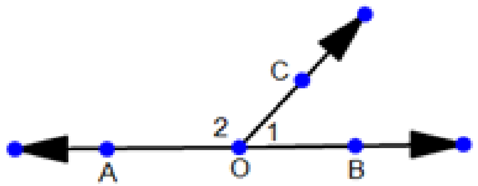
a)



130+40=170. The sum is not 180.

Not supplementary angles.

b)

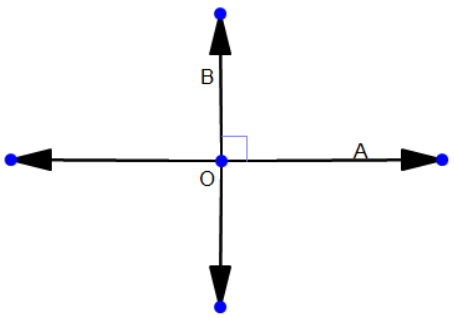


The outer rays form a straight angle.

Supplementary angles.

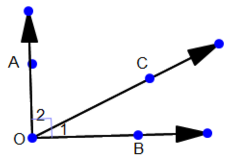
Definition 3

Two lines are perpendicular if they intersect to form a right angle.



The symbol for perpendicular is $\perp$. Thus, $\overleftrightarrow{OA} \perp \overleftrightarrow{OB}$ means "$\overleftrightarrow{OB}$ is perpendicular to $\overleftrightarrow{OA}$". Since segments and rays are parts of lines, then intersecting segments and rays are perpendicular if the lines that contain them are perpendicular.

Sometimes, adjacent angles have their outer rays perpendicular. In the figure at the right, $\overrightarrow{OA} \perp \overrightarrow{OB}$ and $\angle 1$ and $\angle 2$ are acute angles. So $\angle$AOB is a right angle with $m\angle$AOB = 90. By the Angle Addition Postulate $m\angle 1 + m\angle 2 = m\angle$AOB. Therefore, $m\angle 1 + m\angle 2 = 90$, by substituting 90 for $m\angle$AOB.

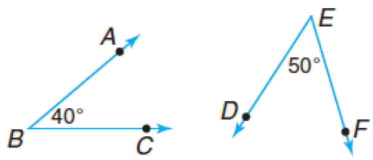


Theorem

If the outer rays of two acute adjacent angles are perpendicular**, then the sum of the measures of the angles is 90.**

Definition 4: Complementary Angles

Two angles are complementary if and only if the sum of their degree measures is 90°



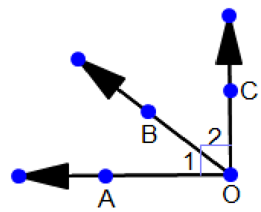
$$m\angle ABC + m\angle DEF = 90^{\circ}$$

Each angle is called a **complement** of the other.

The Theorem above may now be stated as follows:
If the outer rays of two acute adjacent angles are perpendicular, then the angles are complementary.

Example 2: Are the pairs of angles complementary? Explain your answer.

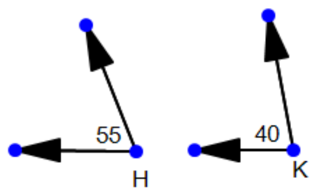
a)



The outer rays form a perpendicular.

Complementary angles

b)



55+40=95. The sum is not 90.

Not complementary angles.

Example 3: Find the measure of a complement and a supplement, if possible.

Angle	Measure	Measure of complement	Measure of supplement
A	35	55	145
B	97	Not possible	83
C (acute angle)	x	90-x	180-x
D (acute angle)	4x-5	90-(4x-5)=95-4x	180-(4x-5)=185-4x

Example 4: The measure of an angle is 60 less than twice the measure of its complement. Find the measure of the angles.

1) **Represent the data:** Let x = measure of complement

$2x - 60$ = measure of the angle
(60 less than twice x)

2) **Write the equation:** $x + (2x - 60) = 90$
Sum of measures of complementary angles is 90.

3) **Solve the equation:**

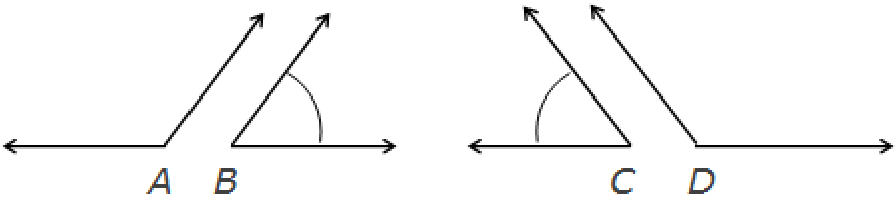
$3x - 60 = 90$
$3x = 150$
$x = 50$

The measure of the complement.
Find the measures of the angle.
$2x - 60 = 2\bullet 50 - 60 = 100 - 60 = 40$

4) **Check the solutions:** Are the angles complementary?
$40 + 50 = 90$✓
Is the angle measure 60 less than twice measure of complement?
$40 = 2\bullet 50 - 60$
$40 = 100 - 60$
$40 = 40$✓

5) **Label the answer:** Thus, the measure of the angles are 40 and 50.

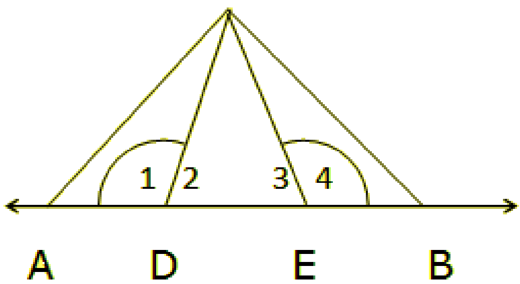
Example 5: Given: ∠B ≅ ∠C,
∠A is a supplement of ∠B, and
∠D is a supplement of ∠C.



Prove: ∠A ≅ ∠D

Statements	Reasons
1. ∠A is a supplement of ∠B ∠D is a supplement of ∠C	1. Given
2. *m*∠A + *m*∠B = 180 *m*∠D + *m*∠C = 180	2.Def of supp ∠s
3. *m*∠A + *m*∠B = *m*∠D + *m*∠C	3. Sub
4. *m*∠B = *m*∠C	
5.∴*m*∠A = *m*∠D(∠A ≅ ∠D)	5. Equations may be subtracted

Example 6: Given: ∠1 ≅ ∠4



Prove: ∠2 ≅ ∠3

Statements	Reasons
1. ∠1 ≅ ∠4	1. Given
2. ∠2 is a supplement of ∠1 ∠3 is a supplement of ∠4	2. Linear pair Post
3. ∴∠2 ≅ ∠3	3. Supp of ≅ ∠s are ≅