Solving Multi-Step Inequalities

An inequality is a statement in which two expressions are not equal.

The following chart shows several inequality symbols and their meaning.

Statements of Inequality	
a is less than b.	$a < b$
a is greater than b.	$a > b$
a is less than or equal to b (or a is at most b).	$a \leq b$
a is greater than or equal to b (or a is at least b).	$a \geq b$
a is greater than b and less than c.	$b < a < c$
a is greater than or equal to b and less than or equal to c.	$b \leq a \leq c$
a is not equal to b.	$a \neq b$

You know that 3 is less than 8. This can be shown in two ways.

$3 < 8$ means 3 is less than 8
$8 > 3$ means 8 is greater than 3

Think about $3 < 8$. This sentence is true. Is $3 > 8$ true? Is $3 = 8$ true? Note that only one of the three sentences is true. This can be summarized by the following property.

Comparison Property

For any two numbers a and b, exactly one of the following sentences is true.

$a < b$ $\quad$ $a = b$ $\quad$ $a > b$

Example 1:

a) Is $4 < 5\frac{1}{2}$ true or false?

$4 < 5\frac{1}{2}$ means 4 is less than $5\frac{1}{2}$. This sentence is true.

b) Is $9 > 4 + 3 + 2 + 1$ true or false?

$$9 \overset{?}{>} 4 + 3 + 2 + 1$$

$$9 \overset{?}{>} 10$$

Since 9 is not greater than 10, the sentence is false.

Recall that there are three properties of equality. Equality is reflexive, symmetric, and transitive. Do inequalities have these same properties?

Reflexive:	$6 > 6$	False
Symmetric:	If $4 > 3$, then $3 > 4$	False
Transitive:	If $6 > 3$ and $3 > 1$, then $6 > 1$	TRUE

The relation > is not reflexive or symmetric. However, it is transitive. Explore some similar examples to verify that < is transitive.

Comparison Property of Order

For all numbers a, b and c,

1) If $a < b$ and $b < c$, then $a < c$.
2) If $a > b$ and $b > c$, then $a > c$.

Addition and Subtraction Properties of Inequalities

For all numbers a, b and c

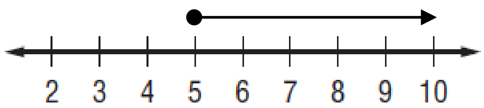
If $a < b$, then $a + c < b + c$.
If $a > b$, then $a + c > b + c$.

If $a < b$, then $a – c < b – c$.
If $a > b$, then $a – c > b – c$.

Example 3: Solve, Graph and check: $m + 7 > 12$

m + 7   2 – 7   Subtract 7 from both sides.
$m \geq 5$

The solution set is {all numbers greater than 5}



Check:
Substitute one or two numbers greater than 5, such as 6 and 10 into the inequality.
For numbers greater than 5, the inequality should be true.

$m + 7 \overset{?}{\geq} 12$	$m + 7 \overset{?}{\geq} 12$
$6 + 7 \geq 12$ Try 6	$10 + 7 \geq 12$ Try 10
$13 \geq 12$✓	$17 \geq 12$✓

What happens if both sides of an inequality are multiplied by the same negative number?

$$-4 < 6$$
$$-4(-3) \overset{?}{<} 6(-3)$$
$$12 \overset{?}{<} -18 \quad \text{False}$$

$$-4 < 6$$
$$-4\left(-\frac{1}{2}\right) \overset{?}{<} 6\left(-\frac{1}{2}\right)$$
$$2 \overset{?}{<} -3 \quad \text{False}$$

The inequality 12 < –18 is false, but 12 > –18 is true. Also, 2 < –3 is false, but 2 > –3 is true. Thus, when both sides of an inequality are multiplied by the same negative number, the direction of the inequality must be reversed.

Multiplication Property of Inequality

For all numbers a, b and c

For c > 0:

If a < b, then ac < bc.
If a > b, then ac > bc.

For c < 0:

If a < b, then ac > bc.
If a > b, then ac < bc.

Recall that $\frac{a}{b}$ (or a ÷ b) is equivalent to $a(\frac{1}{b})$ for all numbers when b is not zero. The multiplication property for inequalities can also apply to division. When solving inequalities, you can multiply (or divide) both sides by the same positive number. You can also multiply (or divide) both sides by the same negative number if you reverse the direction of the inequality.

Division Property of Inequality

For all numbers a, b and c

For c > 0:

If a < b, then $\frac{a}{c} < \frac{b}{c}$.

If a > b, then $\frac{a}{c} > \frac{b}{c}$.

For c < 0:

If a < b, then $\frac{a}{c} > \frac{b}{c}$.

If a > b, then $\frac{a}{c} < \frac{b}{c}$.