Solving Multi-Step Equations

The chart below summarizes the properties that are used to solve simple equations.

The following properties are true for any number *a*, *b*, and *c*.	
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$ $c \neq 0$

Each of these equations can be solved by performing the inverse of the operation indicated in the equation.

$x + 2 = 5$ $3x = 4.8$ $\frac{x}{2} = -11$

But each of the following equations contains more than one operation.

$\frac{x}{4} + 7 = 10$ $\frac{a+10}{3} = 6$ $-5x - 7 = 28$

These equations can be solved using the same methods that were used to solve the first set of equations. However, more than one step is involved.

Recall the order of operations for evaluating an expression. To solve an equation with more than one operation, undo the operations in reverse order.

To solve linear equations of the form: $x \pm a = b$
Do the inverse operation. (Add or subtract a).

To solve linear equations of the form: $ax = b$
Do the inverse operation. (Multiply or divide by a).

To solve linear equations of the form: $ax \pm b = c$
Step 1: Do the inverse operation. (Add or subtract b).
Step2: Do the inverse operation. (Multiply or divide by a).

To solve linear equations of the form: $a(bx + c) = d(ex + f)$
Step1: Use distributive property to get rid of parentheses.
Step2: Collect the terms containing x on one side of the equation and the integers on the other side of the equation.
Step 3: Add the terms containing x.
Step4: Multiply the coefficient of x by its multiplicative inverse.

Step(s) to solve linear equations of the form: $\frac{a}{b}x+c=\frac{d}{e}x+f$

Step1: Clear fractions by multiplying by the LCD of the denominators.
Step2: Collect the terms containing x on one side of the equation and the integers on the other side of the equation.
Step 3: Add the terms containing x.
Step4: Multiply the coefficient of x by its multiplicative inverse.

Example 1: Solve and Check

a) $7=14-3x$

$7=14-3x$	Check:
$7-14=-3x$	$7=14-3\left(\frac{7}{3}\right)$
$-7=-3x$	$7=14-7$
$\frac{-7}{-3}=x$	$7=7$
$\frac{7}{3}=x$	

b) $\frac{d-3}{4}=-3$

$\frac{d-3}{4}=-3$	Check:
$4\bullet\left(\frac{d-3}{4}\right)=-3\bullet(4)$	$\frac{-9-3}{4}=-3$
$d-3=-12$	$\frac{-12}{4}=-3$
$d=-12+3$	$-3=-3$
$d=-9$	

There are some special cases when solving equations:

Case1: Let us solve the following equation $2x+5=2x-3$

$2x+5=2x-3$

$2x-2x=-3-5$

$0=-8$

Is this possible? No, this is a false statement.
This equation has **no solution**.

Case2: Let us solve the following equation $3(x+1)-5=3x-2$

$$3(x+1)-5=3x-2$$
$$3x+3-5=3x-2$$
$$3x-3x=-2+2$$
$$0=0$$

Is this possible? Yes, this is a true statement.

This equation is true for all real numbers. Try to replace any value of x.

This equation is an **identity equation**.

To solve an equation for a certain variable.

Step 1: Collect the terms containing x on one side of the equation and the terms not containing x on the other side of the equation.

Step 2: Use distributive property to write the terms containing x as a product.

Step 3: Divide both sides of the equation by the coefficient of x.