

Solving Equations in Factored Form

Consider the following products.

$$7(0) = 0 \quad -3(0) = 0 \quad (2-2)(3+2)=0 \quad (x+5)(0) = 0$$

The above products all equal zero. Notice that each case **at least one** of the factors is zero.

Zero Product Property

If the product of two quantities equals zero, then at least one of the quantities equals zero.

For all numbers a and b, if $ab = 0$ then $a = 0$ or $b = 0$.

You can use this property to solve equations already in factored form.

Example 1: Use the zero product property to solve the equation. Check your answer.

$$(x+3)(3x-21) = 0$$

$$(x+3)(3x-21) = 0$$

$$x+3=0 \text{ or } 3x-21=0 \quad \text{Use the zero product property}$$

$$x=-3 \text{ and } 3x=21 \Rightarrow x=7 \quad \text{Solve each equation}$$

$$\text{The solution set} = \{-3, 7\}$$

Check

$$\text{If } x = -3, \text{ then } (-3+3)(3 \cdot (-3) - 21) = 0 \Rightarrow (0)(-30) = 0 \Rightarrow 0 = 0 \text{ True}$$

$$\text{If } x = 7, \text{ then } (7+3)(3 \cdot (7) - 21) = 0 \Rightarrow (10)(0) = 0 \Rightarrow 0 = 0 \text{ True}$$

Example 2: Write an equation in factored form if the solution set is $\{0, 1, -4\}$.

$$x=0, x=1, x=-4 \quad \text{Write the factor for each solution.}$$

$$(x-0)(x-1)(x-(-4)) \quad \text{Each factor is } (x - \text{solution})$$

$$= x(x-1)(x+4)$$

Example 3: Tala gave this puzzle to her friends. "The product of 4 times my age and 45 less than 3 times my age is zero. How old am I?"

EXPLORE This problem can be solved by using an equation and zero product property.
Let y = Tala's age

PLAN $4y(3y - 45) = 0$

SOLVE $4y = 0$ or $3y - 45 = 0$
 $y = 0$ $3y = 45 \Rightarrow y = 15$

EXAMINE Although 0 is a solution of the equation $4y(3y - 45) = 0$, Tala cannot be 0 years old, so this solution is rejected. Therefore, Tala is 15 years old.