Remarkable Identities

Consider the following products.

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

$$(2+3)^2 = 5^2 = 25 \qquad (2+3)^2 = (2+3)(2+3) = 4+6+6+9 = 25$$

Using a similar procedure, the general form $(a + b)^2$ can be simplified as follows.

Square of a Sum

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(\textit{first term})^2 + \textit{twice the product of the first term and the second term} + (\textit{second term})^2$$

Example 1: Expand

a) $(x+3)^2$

$$(x+3)^2$$
$$= x^2 + 2 \bullet x \bullet 3 + 3^2$$
$$= x^2 + 6x + 9$$

b) $(4x+5y)^2$

$$(4x+5y)^2$$
$$= (4x)^2 + 2 \bullet 4x \bullet 5y + (5y)^2$$
$$= 16x^2 + 40xy + 25y^2$$

Square of a Difference

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(\textit{first term})^2 - \textit{twice the product of the first term and the second term} + (\textit{second term})^2$$

Consider the following products.

$$(a+b)(a-b) = a^2 + ab - ab - b^2 = a^2 - b^2$$
$$(6+3)(6-3) = 9 \bullet 3 = 27 \qquad (6+3)(6-3) = 6^2 - 3^2 = 36 - 9 = 27$$

Product of a Sum and a Difference

$$(a+b)(a-b) = a^2 - b^2$$

$$(\textit{first term})^2 - (\textit{second term})^2$$