Rational Numbers

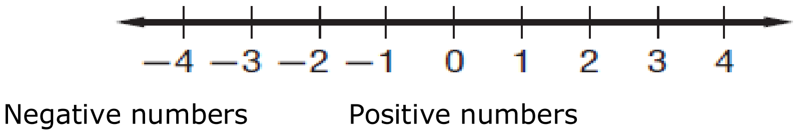
Different sets of numbers can be represented by lists of their members. For example:

Natural or counting numbers: N = $\{1, 2, 3, 4...\}$

Whole numbers: W = $\{0, 1, 2, 3, 4...\}$

Integers: Z = $\{... -4, -3, -2, -1, 0, 1, 2, 3, 4...\}$

The sets N, W, and Z, can also be shown on a number line. Remember that 0 is known as the origin and is neither positive, nor negative. Numbers to the right of 0 are **positive**, and those to the left of 0 are **negative**.



Although the labeled numbers are typically integers, there are many numbers on a number line that are not integers. Numbers like $\frac{2}{3}$, $-\frac{4}{15}$ and $\frac{25}{5}$ are included on a number line and are called **rational numbers.**

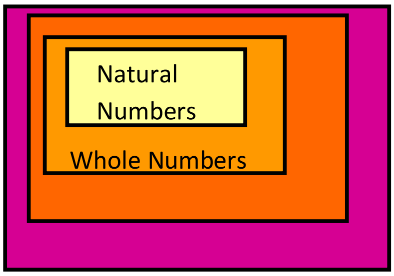
Definition of a Rational Number

A rational number is any number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and b is not equal to 0.

The set of rational numbers cannot be listed as the integers can be, but this set does include the integer since any integer, *a* can be expressed as $\frac{a}{1}$.

Any rational number can be written as either a terminating decimal or a repeating decimal. Repeating decimals do not end: they continuously repeat a pattern of digits to the right of the decimal. Terminating decimals simply end at a certain point.

The **Venn diagram** at the right show how all the sets of numbers discussed relate to one other.



The number line represents the set of rational numbers. Each point on the number line represents exactly one number. Each number corresponds with exactly one point on the number line. This allows the number on the number line to order. Numbers on the number line increase in value from left to right.

<u>The symbols used for ordering are as follows:</u>

< less than ≤ less than or equal to > greater than ≥ greater than or equal to
= equal to ≠ not equal to

Example 1: Insert an ordering symbol to make each statement true

a. 5___ – 7

5 is to the right of –7 on a n umber line, so 5 > –7.

b. $\frac{1}{2}$ ___ $\frac{8}{16}$

$\frac{1}{2}$ and $\frac{8}{16}$ represent the same number, so $\frac{1}{2} = \frac{8}{16}$

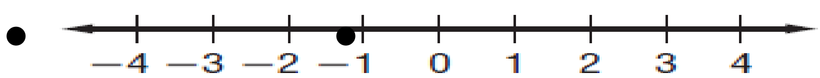
c. –4.8____ –4.7
–4.8 is to the left of –4.7 on a number line, so –4.8 < –4.7

d. $4\frac{2}{3}$_____ 6

$4\frac{2}{3}$ is to the left of 6 on a number line, so $4\frac{2}{3} < 6$.

Opposites and Absolute Value

On the number line, two numbers that lie on opposite sides of 0 and are the same distance from 0 are **opposites.** The opposite of 2 is negative 2, written as –2. The opposite of a positive number is always a negative number.



As you can see from the number line, the opposite to –2 is +2. In fact, the opposite of a negative number is always a positive number. This is represented with symbols as follows:

$$-(-2) = 2$$

Opposite of zero is zero itself. –0 = 0

Opposite numbers are helpful in understanding other important concepts in algebra, such as absolute value.

Definition of Absolute Value
The absolute value of a real number x is the distance from x to 0 on a number line. **The symbol $\lvert x \rvert$ means absolute value of x.**

Notice that a number and its opposite have the same absolute value. The absolute value of 2 and –2 are the same. Each is 2 units from 0.

The absolute value of a positive number or 0 is the same as the number itself.

The absolute value of a negative number is the opposite of the number.