Polynomials

A polynomial is a monomial, or a sum of monomials.
The expression $x^3 + 6x^2 + 12x + 8$ is an example of a polynomial in one variable, x.

Some polynomial expressions have special names that are determined either by their degree or by the number of terms, as illustrated in the table.

Polynomial	# of terms	Name by # of terms	Degree	Name by degree
12	1	monomial	0	constant
$8x$	1	monomial	1	linear
$4x^2 + 3$	2	binomial	2	quadratic
$5x^3 + x^2$	2	binomial	3	cubic
$3x^2 - 4x + 6$	3	trinomial	2	quadratic
$3x^4 - 4x^3 + 6x^2 + 7$	4	polynomial	4	quadric

Example 1: State whether each expression is a polynomial. If the expression is a polynomial, identify it as a monomial, binomial, or trinomial.

a) $8x^2 - 3xy$

The expression $8x^2 - 3xy$ can be written as $8x^2 + (-3xy)$.
Therefore, $8x^2 - 3xy$ is a polynomial because it can be written as the sum of two monomials, $8x^2$, and $-3xy$. Since it has two terms, $8x^2 - 3xy$ is a binomial.

b) $\frac{5}{2y^2} - 7y + 6$

The expression $\frac{5}{2y^2} - 7y + 6$ is not a polynomial because $\frac{5}{2y^2}$ is not a monomial.

c) $3x^2 + 2x + 4$
The expression $3x^2 + 2x + 4$ is a polynomial because it is the sum of three monomials, $3x^2$, $2x$, and 4. Since it has three terms, $3x^2 + 2x + 4$ is a trinomial.

The **degree** of a polynomial in one variable is determined by the exponent with the greatest value within the polynomial. The degree of $9 - 4x^2$ is 2.
To find the degree of a polynomial, first find the degree of each of its terms.
The degree of the polynomial is the **greatest** of the degrees of the terms.

The terms of a polynomial are usually arranged so that the powers of one variable are in either ascending or descending order.

Ascending order	Descending order
$3 + 5a - 8a^2 + a^3$	$a^3 - 8a^2 + 5a + 3$
(in x) $5xy + x^3y^2 - x^4 + x^5y^2$	(in x) $x^5y^2 - x^4 + x^3y^2 + 5xy$
(in y) $x^3 - 3x^2y + 4x^2y^2 - y^3$	(in y) $-y^3 + 4x^2y^2 - 3x^2y + x^3$

However in the ***standard form***, the terms of a polynomial are ordered from left to right in the descending order, which means from the greatest exponent to the least.