Perfect Squares and Factoring

Square of a Sum

$(a+b)^2 = a^2 + 2ab + b^2$

$(\textit{first term})^2 + \textit{twice the product of the first term and the second term} + (\text{second term})^2$

Square of a Difference

$(a-b)^2 = a^2 - 2ab + b^2$

$(\textit{first term})^2 - \textit{twice the product of the first term and the second term} + (\text{second term})^2$

You can use these patterns to factor certain polynomials. **For example:**

Multiplication Form	Factorization Form
$(x+3)^2 = (x+3)(x+3) = x^2 + 6x + 9$	$x^2 + 6x + 9 = (x+3)^2$

Rule: A trinomial is a perfect square if:

1) The first and last terms are perfect squares.

2) The middle term is two times one factor from the first term and one factor from the last term.

Example 1: Determine if each of the following is a perfect square. If it is, factor it.

1) $16a^2 + 72a + 81$

Is the first term a square? $16a^2 = (4a)^2$

Is the last term a square? $81 = (9)^2$

Is the middle term twice the product of 4a and 9? $72a = 2(4a)(9)$

$16a^2 + 72a + 81$ is a perfect square.

$16a^2 + 72a + 81 = (4a)^2 + 2(4a)(9) + (9)^2 = (4a + 9)^2$

2) $4a^2 - 20a + 15$

Is the first term a square? $4a^2 = (2a)^2$

Is the last term a square? 15 is not a perfect square number.

$4a^2 - 20a + 15$ is not a perfect square.

3) $16x^2 - 26x + 49$

Is the first term a square? $16x^2 = (4x)^2$

Is the last term a square? $49 = (7)^2$

Is the middle term 2 times the product of 4x and 7? $26x \neq 2(4x)(7)$

$16x^2 - 26x + 49$ is not a perfect square.

4) $x^2 - 20x - 100$

Is the first term a square? $x^2 = (x)^2$ Yes

Is the last term a square? $-100 \neq (-10)^2$

$x^2 - 20x - 100$ is not a perfect square.