Monomials

Definition: **A monomial** is a number, a variable, or a product of a number and one or more variables.

These are monomials., $-7, y, 7k, -2x^3, \frac{1}{4}ab^2c^3d^4$

These are not monomials. $a+b, \frac{x}{y}, 7-2x^3, \frac{1}{x^2}, \frac{3m}{4n}$

Note: **Monomials** that are real numbers are called **constants**.

Rule: The **degree** of a monomial is the sum of the exponents of its variables. The real number multiplied by the variable is called the **coefficient.**

Example 1: Determine whether each expression is a monomial. If it is, find its coefficient and degree.

Expression	Monomial or not	Coefficient	Degree
x	Monomial	1	1
$3x^5$	Monomial	3	5
$-2x^3y$	Monomial	-2	3+1=4
$\frac{2}{3}a^3b^2c^4d$	Monomial	$\frac{2}{3}$	3+2+1+4=10
$\frac{1}{x^2}$	Not monomial		
$10+2x^3$	Not monomial		
$\frac{2x^2}{5y^3}$	Not monomial		

Consider the following statements.

$4 \bullet 8 = 32 \qquad 2 \bullet 32 = 64$

$2^2 \bullet 2^3 = 2^5 \qquad 2^1 \bullet 2^5 = 2^6$

These examples suggest that you can multiply powers that have the same base by adding their exponents.

Product of Powers

$a^m \bullet a^n = a^{m+n}$ **where a is any real number and m and n are integers**

Take a look at the example below.

$\left(5^2\right)^3 = 5^2 \bullet 5^2 \bullet 5^2 = 5^6$

Since $\left(5^2\right)^3 = 5^6$, the example suggests that you can find the power of a power by multiplying exponents

Power of a Power

$\left(a^m\right)^n = a^{mn}$ **where a is any real number and m and n are integers.**

Take a look at the example below.

$(xy)^3 = (xy)(xy)(xy) = (x \bullet x \bullet x)(y \bullet y \bullet y) = x^3y^3$

Power of a Product

$(ab)^m = a^m b^m$ **where a is any real number and m is an integer**

Power of a monomial

$\left(a^m b^n\right)^p = a^{mp} b^{np}$ **where is any real number and m, n and p are integers.**

Consider the following quotients.

$\frac{32}{8} = 4 \quad \frac{64}{4} = 16$

$\frac{2^5}{2^3} = 2^2 \quad \frac{2^6}{2^2} = 2^4$

Quotients of Powers

$\frac{a^m}{a^n} = a^{m-n}$ **where a is any real number and m is an integer**

Consider the following quotients.

$$\frac{2^3}{2^3} = \frac{\cancel{2} \bullet \cancel{2} \bullet \cancel{2}}{\cancel{2} \bullet \cancel{2} \bullet \cancel{2}} = 1 \qquad \frac{2^3}{2^3} = 2^0$$

Zero Exponent

For any non-zero real number a, $a^0 = 1$

Note: 0^0 is not defined

Study the two ways shown below.

$$\frac{2^2}{2^5} = \frac{\cancel{2} \bullet \cancel{2}}{\cancel{2} \bullet \cancel{2} \bullet 2 \bullet 2 \bullet 2} = \frac{1}{2^3} \qquad \frac{2^2}{2^5} = 2^{-3}$$

Negative Exponents

For any non-zero real number a, $a^{-m} = \frac{1}{a^m}$