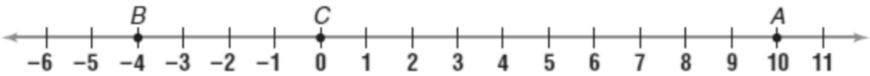
Measuring and Constructing Segments

The number that corresponds to a point on a number line is called the coordinate of the point. On the number line below, 10 is the coordinate of point *A*. The coordinate of point *B* is -4. Point *C* has coordinate 0 and is called the origin.



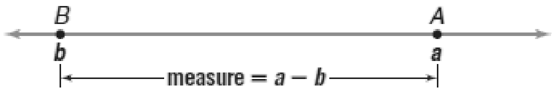
The distance between two points *A* and *B* on a number line is found by using the Distance and Ruler Postulates.

Postulate 1: Distance Postulate

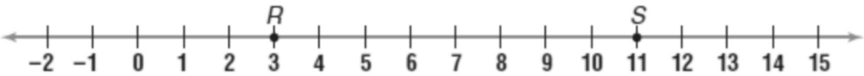
For any two points on a line and a given unit of measure, there is a unique positive real number called the measure of the distance between the points.

Postulate 2: Ruler Postulate

Points on a line are paired with real numbers, and the measure of the distance between two points is the positive difference of the corresponding numbers



Suppose you want to find the distance between points *R* and *S* on the number line below.

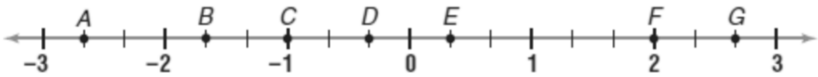


The measure of the distance between points *R* and *S* is the positive difference 11 - 3, or 8. The notation for the measure of the distance between two points is indicated by the capital letters representing the points. Since the measure from point *S* to point *R* is the same as from *R* to *S*, you can write *RS* = 8 or *SR* = 8.

Another way to calculate the measure of the distance is by using absolute value. The absolute value of a number is the number of units a number is from zero on the number line. In symbols, the absolute value is denoted by two vertical slashes.

$SR = |11 - 3| = |8| = 8$ $\qquad$ $RS = |3 - 11| = |-8| = 8$

Example 1: Use the number line to find the measure of the indicated segment.



a) CF

$$CF = |2-(-1)| = |2+1| = |3| = 3$$

b) AE

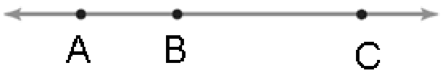
$$AE = \left|\frac{1}{3} - \left(-2\frac{2}{3}\right)\right| = \left|\frac{1}{3} - \left(-\frac{8}{3}\right)\right| = \left|\frac{1}{3} + \frac{8}{3}\right| = \left|\frac{9}{3}\right| = 3$$

Given three collinear points on a line, one point is always between the other two points. In the figure below, point *B* is between points *A* and *C*.

Point *B* lies to the right of point *A* and to the left of point *C*.

Postulate 3: Segment Addition Postulate

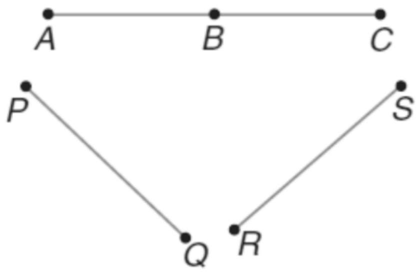
If points A, B and C are three collinear points such that point B is between points A and C, then $AB + BC = AC$



Definition 1: Congruent Segments

Two segments are congruent if and only if they have the same length

In the figures, $\overline{AB}$ is congruent to $\overline{BC}$, and $\overline{PQ}$ is congruent to $\overline{RS}$. The symbol $\cong$ is used to represent congruence.



$\overline{AB} \cong \overline{BC}$ and $\overline{PQ} \cong \overline{RS}$

From the definition of congruent segments, we can also say $AB = BC$ and $PQ = RS$ P

Since congruence is related to the equality of segment measures, there are properties of congruence that are similar to the corresponding properties of equality. These statements are called theorems. Theorems are statements that can be justified by using logical reasoning.

We know that *AB* = *AB*. Therefore, $\overline{AB} \cong \overline{AB}$ and we can see that congruence is reflexive. You can make similar arguments to show congruence is symmetric and transitive.

Theorem 1

Congruence of segments is reflexive

$$\overline{AB} \cong \overline{AB}$$

Theorem 2

Congruence of segments is symmetric

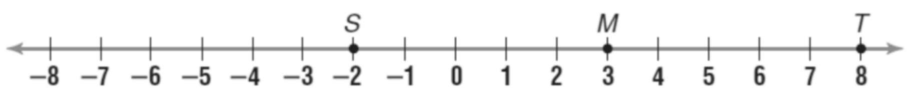
If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$

Theorem 3:

Congruence of segments is transitive

If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$

There is a unique point on every segment called the midpoint. On the number line below, *M* is the midpoint of $\overline{ST}$. What do you notice about *SM* and *MT*?



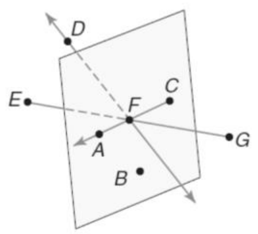
Definition 2: Midpoint of a segment

A point M is the midpoint of a segment $\overline{ST}$ **if and only if M is between S and T and SM=MT**

M is the midpoint of $\overline{ST} \Rightarrow \overline{SM} \cong \overline{MT}$ or $SM = MT$

The midpoint of a segment separates the segment into two segments of equal length. So, by the definition of congruent segments, the two segments are congruent.

To bisect something means to separate it into two congruent parts.
The midpoint of a segment bisects the segment because it separates the segment into two congruent segments. A point, line, ray, segment, or plane can also bisect a segment.



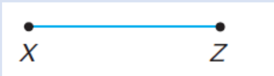
Point F bisects $\overline{EG}$.
$\overrightarrow{FD}$ bisects $\overline{EG}$.
$\overleftrightarrow{FA}$ bisects $\overline{EG}$.
$\overline{AC}$ bisects $\overline{EG}$.
Plane ABC bisects $\overline{EG}$.

The midpoint of the segment must be found to separate a segment into two congruent segments. If the segment is part of a number line, you can use arithmetic to find the midpoint. If there is no number line, you can use a construction to find the midpoint.

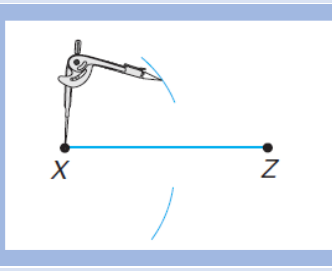
Hands On Construction

Materials: compass straightedge

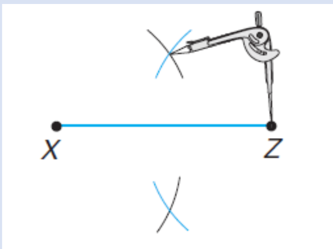
Step 1: Use a straightedge to draw the segment you wish to bisect. Name it XZ.



Step 2: Place the compass at point X. Use any compass setting greater than one half of XZ. Draw an arc above and below XZ.



Step 3: Using the same compass setting, place the compass at point Z.
Draw an arc above and below XZ.
These arcs should intersect the ones previously drawn.



Step 4: Use a straightedge to align the two intersections. Draw a segment that intersects XZ. Label the point of intersection Y.