## Measuring and Constructing Angles

Opposite rays are two rays that are part of the same line and have only their endpoints in common.


## $\overrightarrow{X Y}$ and $\overrightarrow{X Z}$ are opposite rays.

The figure formed by opposite rays is also referred to as a straight angle.
There is another case where two rays can have a common endpoint. This figure is called an angle.
Some parts of angles have special names. The common endpoint is called the vertex, and the two rays that make up the angle are called the sides of the angle.


There are several ways to name the angle shown above:

1. Use the vertex and a point from each side: $\angle R S T$ or $\angle T S R$

The vertex letter is always in the middle.
2. Use the vertex only: $\angle S$

If there is only one angle at a vertex, then the angle can be named with that vertex.
3. Use a number: $\angle 1$

## Definition 1: Angle

An angle is a figure formed by two noncollinear rays that have a common endpoint

$\angle 2$
$\angle E$
$\angle D E F$
$\angle F E D$

## Mathelpers

## Example 1: Name the angle shown below in four different ways



The vertex is H and the sides are $\overrightarrow{H G}$ and $\overrightarrow{H I}$ The angle can be named in four different ways $\angle H, \angle 3, \angle G H I$ and $\angle I H G$

An angle separates a plane into three parts: the interior of the angle, the exterior of the angle, and the angle itself. In the figure shown,

point $W$ and all other points in the blue region are in the interior of the angle. Point $V$ and all other points in the yellow region are in the exterior of the angle. Points $X, Y$, and $Z$ are on the angle.

## Postulate 1: Angle Measure Postulate

For every angle, there is a unique positive number between 0 and 180 called the degree measure of the angle


$$
m \angle A B C=n
$$

$$
0^{\circ}<n<180^{\circ}
$$

In geometry, angles are measured in units called degrees. The symbol for degree is ${ }^{\circ}$. The angle shown measures 75 degrees.


$$
m \angle P Q R=75
$$

You can use a protractor to measure angles and sketch angles of given measure. Use a protractor to measure $\angle D E F$.

Step 1: Place the center point of the protractor on vertex E. Align the straightedge with side EF.


Step 2: Use the scale that begins with 0 at $E F$. Read where the other side of the angle, $E D$, crosses this scale. $m \angle D E F=125$

$\angle D E F$ measures $125^{\circ}$

Once the measure of an angle is known, the angle can be classified as one of three types of angles. These types are defined in relation to a right angle.


A right angle is an angle whose measure is $90^{\circ}$


An acute angle is an angle whose measure is less than $90^{\circ}$


An obtuse angle is an angle whose measure is greater than $90^{\circ}$ and less than $180^{\circ}$

## Mathelpers

To construct two angles of the same measure requires a compass and straightedge.

## Hands On Construction



Step 1: Draw an angle like $\angle \mathbf{P}$ on your paper.

Step 2: Use a straightedge to draw a ray on your paper. Label its endpoint $T$.


Step 3: With P as the center, draw a large arc that intersects both sides of $\angle P$. Label the points of intersection $Q$ and $R$.

Step 4: Using the same compass setting, put the compass at point $T$ and draw a large arc that starts above the ray and intersects the ray.

Label the point of intersection S.

Step 5: Place the point of the compass on $R$ and adjust so that the pencil tip is on Q.


Step 6: Without changing the setting, place the compass at point S and draw an arc to intersect the larger arc you drew in Step 4.

Label the point of intersection $U$.

Step 7: Use a straightedge to draw TU.


## Postulate 2: Angle Addition Postulate

For any angle PQR , if A is in the interior of $\angle \mathrm{PQR}$, then $m \angle P Q A+m \angle A Q R=m \angle P Q R$


$$
m \angle 1+m \angle 2=m \angle P Q R
$$

Just as every segment has a midpoint that bisects the segment, every angle has a ray that bisects the angle. This ray is called an angle bisector.

## Definition 2: Angle Bisector

The bisector of an angle is the ray with its endpoint at the vertex of the angle, extending into the interior of the angle. The bisector separates the angle into two angles of equal measure

$\overrightarrow{P W}$ is the bisector of $\angle R P Q$

$$
\angle R P W \cong \angle W P Q
$$

$$
m \angle R P W=m \angle W P Q
$$

To construct the angle bisector requires a compass and straightedge.

## Hands On Construction

Step 1 Draw an angle like $\angle \mathrm{A}$ on your paper.

Step 2 Place a compass at point A and draw a large arc that intersects both sides of $\angle \mathbf{A}$. Label the points of intersection $B$ and $C$.


Step 3 With the compass at point B, draw an arc in the interior of $\angle \mathbf{A}$.

Step 4 Keeping the same compass setting, place the compass at point $C$. Draw an arc that intersects the arc drawn in Step 3. Label the point of intersection $D$.

Step 5 Draw AD.


Definition
Two angles are congruent if and only if they have the same degree measure

$\angle A \cong \angle B$ if and only if $m \angle A=m \angle B$

