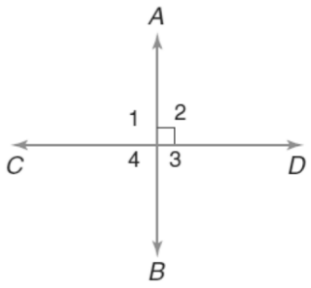
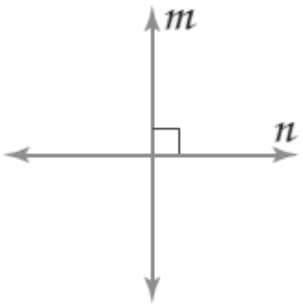
Lines Relationships

Lines that intersect at an angle of 90 degrees are perpendicular lines. In the figure below, lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are perpendicular.



Definition 1: Perpendicular Lines

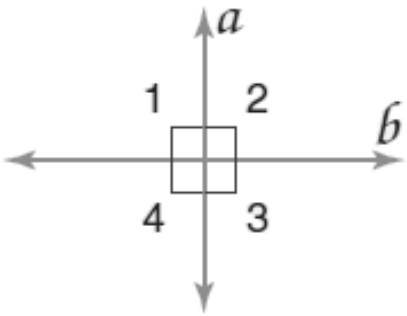
Perpendicular lines are lines that intersect to form a right angle



$m \perp n$

Theorem 1

If two lines are perpendicular, then they form four right angles



$a \perp b$

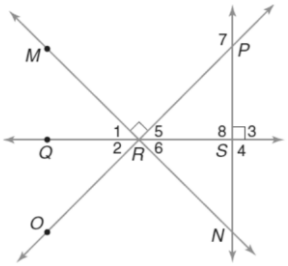
$m\angle 1 = 90^\circ$

$m\angle 2 = 90^\circ$

$m\angle 3 = 90^\circ$

$m\angle 4 = 90^\circ$

Example 1: In the figure, $\overleftrightarrow{OP} \perp \overleftrightarrow{MN}$ and $\overleftrightarrow{NP} \perp \overleftrightarrow{QS}$. Determine whether each of the following is true or false. Justify.



a) $\angle PRN$ is an acute angle

False. Since $\overleftrightarrow{OP} \perp \overleftrightarrow{MN}$, $\angle PRN$ is a right angle

b) $\angle 4 \cong \angle 8$

True. $\angle 4$ *and* $\angle 8$ are vertical angles and vertical angles are congruent

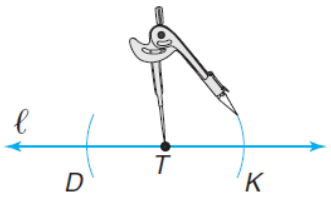
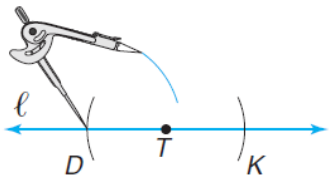
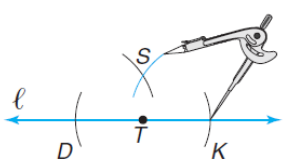
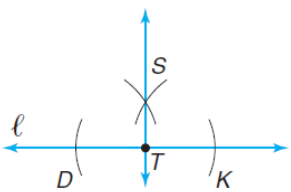
c) $m\angle 6 + m\angle 5 = 90^\circ$

$\overleftrightarrow{OP} \perp \overleftrightarrow{MN}$, so $\angle 5$ *and* $\angle 6$ are complementary angles

To construct a line perpendicular to a given line using a compass and straightedge

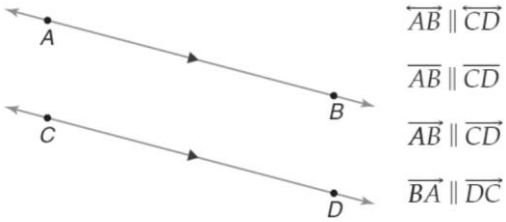
Hands On Construction

Materials: compass straightedge

Step 1 Draw a line l that contains a point *T*.	
Step 2 Place the compass at point *T*. Using the same compass setting, draw arcs to the left and right of *T*, intersecting line l. Label these points *D* and *K*.	
Step 3 Open the compass to a setting greater than *DT*. Put the compass at point *D* and draw an arc above line l.	
Step 4 Using the same compass setting, put the compass at point *K* and draw an arc to intersect the one previously drawn. Label the point of intersection *S*.	
Step 5 Use a straightedge to draw *ST*.	

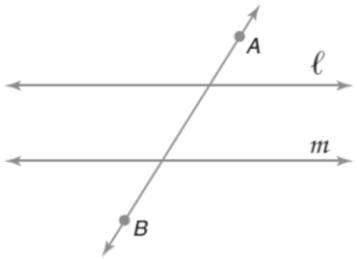
Definition 2

Two lines are parallel if and only if they are in the same plane and do not intersect



Since segments and rays are parts of lines, they are considered parallel if the lines that contain them are parallel.

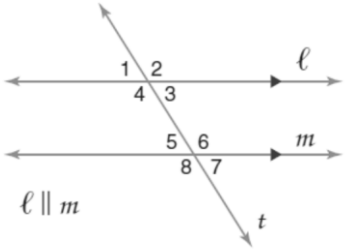
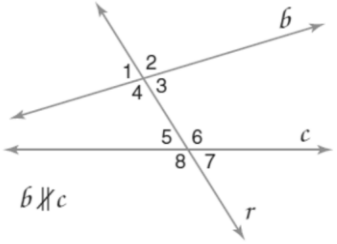
In geometry, a line, line segment, or ray that intersects two or more lines at different points is called a transversal. *AB* is an example of a transversal. It intersects lines l and *m*. Note all of the different angles that are formed at the points of intersection.



Definition 3

In a plane, a line is a transversal if and only if it intersects two or more lines, each at a different point.

The lines cut by a transversal may or may not be parallel.

Parallel Lines	Nonparallel Lines
	
t is a transversal for l and m	**r is a transversal for b and c**

When a transversal intersects two lines, eight angles are formed, as shown in the figures above. These angles are given special names.

Interior Angles: Lie between the two lines $\angle 3, \angle 4, \angle 5, \angle 6$

Exterior Angles: Lie outside the two lines $\angle 1, \angle 2, \angle 7, \angle 8$

Alternate interior angles: are on opposite sides of the transversal

$\angle 3$ *and* $\angle 5$, $\angle 6$ *and* $\angle 4$

Alternate exterior angles: are on opposite sides of the transversal

$\angle 1$ *and* $\angle 7$, $\angle 2$ *and* $\angle 8$

Consecutive interior angles (same sided angles): are on the same side of the transversal

$\angle 3$ *and* $\angle 6$, $\angle 5$ *and* $\angle 4$