Laws and Exponents

Notice what happens when you multiply two powers with the same base.

$$a^4 \bullet a^3 = \underbrace{\overbrace{(a \bullet a \bullet a \bullet a)}^{\text{4 factors}} \bullet \overbrace{(a \bullet a \bullet a)}^{\text{3 factors}}}_{\text{7 factors}} = a^{4+3} = a^7$$

This example suggests a rule for multiplying powers with the same base.

Product of Powers Property

Words: To multiply powers with the same base, add their exponents.

Algebra: $a^m \bullet a^n = a^{m+n}$

Numbers: $4^3 \bullet 4^2 = 4^{3+2} = 4^5$

Example 1: Lake Powell, the reservoir behind the Glen Canyon Dam in Arizona can hold about 10^{12} cubic feet of water when full.
There are about 10^{27} water molecules in 1 cubic foot of water. About how many water molecules can the reservoir hold?

Number of water molecules in reservoir	=	Cubic feet of water in reservoir	•	Number of water molecules in a cubic foot

$= 10^{12} \bullet 10^{27}$ Substitute values.
$= 10^{12+27}$ Product of powers property.
$= 10^{39}$ Add exponents.

Lake Powell can hold about 10^{39} molecules of water.

There is a related rule you can use for dividing powers with the same base. The following example suggests this rule.

$$\frac{a^5}{a^2} = \frac{\overbrace{a \bullet a \bullet a \bullet a \bullet a}^{\text{5 factors}}}{\underbrace{a \bullet a}_{\text{2 factors}}} = \frac{a \bullet a \bullet a \bullet \cancel{a}^1 \bullet \cancel{a}^1}{\cancel{a}_1 \bullet \cancel{a}_1} = \underbrace{a \bullet a \bullet a}_{\text{3 factors}} = a^{5-2} = a^3$$

Quotient of Powers Property

Words: To divide powers with the same base, subtract the exponent of the denominator form the exponent of the numerator.

Algebra: $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Numbers: $\frac{6^8}{6^5} = 6^{8-5} = 6^3$

Consider the following pattern of powers of 2.

$$2^3 = 8$$
$$2^2 = 4$$
$$2^1 = 2$$
$$2^0 = ?$$
$$2^{-1} = ?$$
$$2^{-2} = ?$$

By extending the pattern, you can conclude that $2^0 = 1$, $2^{-1} = \frac{1}{2}$, and $2^{-2} = \frac{1}{4}$. Because $\frac{1}{2} = \frac{1}{2^1}$ and $\frac{1}{4} = \frac{1}{2^2}$, the pattern suggests the following definitions for negative and zero exponents.

Negative and Zero Exponents

For any nonzero number a, $a^0 = 1$.

For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$.

Rewriting Fractions You can use the prime factorization of a number to write a fraction as an expression involving negative exponents.

Products and Quotients of Powers You can use the product of powers property and the quotient of powers property to find products and quotients that involve negative exponents.

Scientific notation

The retina is a layer of the eyeball that contains rods and cones. Rods and cones are cells that absorb light and change it to electric signals that are sent to the brain. The human retina is about 0.00012 meter thick and contains about 120,000,000 rods and about 6,000,000 cones.

You can use scientific notation to write these numbers. Scientific notation is a shorthand way of writing numbers using powers of 10.

Using Scientific Notation

A number is written in scientific notation if it has the form $c \times 10^n$ where $1 \le c \le 10$ and n is an integer.

Standard form	Product form	Scientific notation
725,000	7.25 × 100,000	7.25×10^5
0.006	6 × 0.001	6×10^{-3}