Factoring by Grouping

Some polynomials have four terms. It may be possible to factor these polynomials by first grouping the terms in pairs and then factor out the GCF from each group. Then use the distributive property again to factor out the common binomial.

Example 1: Factor by grouping $12a^3 - 9a^2 + 20a - 15$. Check your answer.

$12a^3 - 9a^2 + 20a - 15$

$= (12a^3 - 9a^2) + (20a - 15)$ Group terms.

$= 3a^2(4a - 3) + 5(4a - 3)$ Factor out the GCF for each group.

$= (4a - 3)(3a^2 + 5)$ Factor out the GCF.

Check

$(4a - 3)(3a^2 + 5) = 12a^3 - 9a^2 + 20a - 15$

Example 2: Factor each polynomial by grouping.

1) $3x^3 - 15x^2 + 10 - 2x$

$3x^3 - 15x^2 + 10 - 2x$

$= (3x^3 - 15x^2) + (10 - 2x)$ Group terms.

$= 3x^2(x - 5) + 2(5 - x)$ Factor out the GCF for each group.

$= 3x^2(x - 5) - 2(x - 5)$ Notice that $(x - 5)$ *and* $(5 - x)$ are opposites. $(x - 5) = -(x - 5)$.

$= (x - 5)(3x^2 - 2)$

Sometimes you can group the terms in more than one way.

$3x^3 - 15x^2 + 10 - 2x$

$= (3x^3 - 2x) + (10 - 15x^2)$

$= x(3x^2 - 2) + 5(2 - 3x^2)$

$= x(3x^2 - 2) - 5(3x^2 - 2)$

$= (3x^2 - 2)(x - 5)$

2) $9x^3 + 18x^2 - x - 2$

$9x^3 + 18x^2 - x - 2$

$= (9x^3 + 18x^2) + (-x - 2)$ Group terms.

$= 9x^2(x + 2) - (x + 2)$ Factor out the GCF for each group. Notice that the GCF of the second group is -1.

$= (9x^2 - 1)(x + 2)$ Factor out the GCF.

$= (3x - 1)(3x + 1)(x + 2)$ Notice that $(9x^2 - 1)$ can be factored using difference of two squares.