Factoring Differences of Squares

The product of the sum of 2 numbers by their difference is equal to the difference of their squares.

Product of a Sum and a Difference

$(a+b)(a-b)=a^2-b^2$

$(\ first\ term)^2-(\text{second } term)^2$

For example:

Multiplication Form	Factorization Form
$(x+3)(x-3)=x^2-9$	$x^2-9=(x+3)(x-3)$

Note: To factor difference of 2 squares, check the following:
1) It is difference.
2) The numbers are perfect square numbers.
A perfect square number is a number that can be written in the form a^2.

The following table shows some perfect square numbers.

Perfect square numbers	Reason
1	1^2
4	2^2
9	3^2
16	4^2
25	5^2
36	6^2
49	7^2
64	8^2
81	9^2
100	10^2
121	11^2
144	12^2

Example 1: Determine whether each binomial is a difference of two squares. If so, factor it. If not, explain.

1) a^2-64

a^2-64
$=a^2-8^2$
$=(a+8)(a-8)$

2) $1-9x^2$

$1-9x^2$
$=1^2-(3x)^2$
$=(1-3x)(1+3x)$

3) $\frac{1}{4}m^2-\frac{9}{25}x^2$

$\frac{1}{4}m^2-\frac{9}{25}x^2$
$=\left(\frac{1}{2}m\right)^2-\left(\frac{3}{5}x\right)^2$
$=\left(\frac{1}{2}m+\frac{3}{5}x\right)\left(\frac{1}{2}m-\frac{3}{5}x\right)$

4) x^2+25

x^2+25 Cannot be factored because it is a sum of squares and not a difference.

5) $x-100$

$x-100$ Cannot be factored because x is not a perfect square number

6) y^2-10

y^2-10 Cannot be factored because 10 is not a perfect square number.

Sometimes the terms of a binomial have common factors. If so, the GCF should always be factored out before factoring the difference of squares.
Occasionally, factoring the difference of squares needs to be used more than once in a problem.

Example 2: Factor completely

1) $36y^2-16x^2$

1)$36y^2-16x^2$ The GCF is 4.
$=4\left(9y^2-4x^2\right)$ Factor out 4.
$=4(3y-2x)(3y+2x)$ Factor difference of squares.

2) $81a^4-16b^4$

2)$81a^4-16b^4$ The GCF is 1.
$=\left(9a^2-4b^2\right)\left(9a^2+4b^2\right)$ Factor difference of squares. Note that $a^4=\left(a^2\right)^2$
$=(3a-2b)(3a+2b)\left(9a^2+4b^2\right)$ Factor difference of squares. Note that $9a^2+4b^2$ cannot be factored.