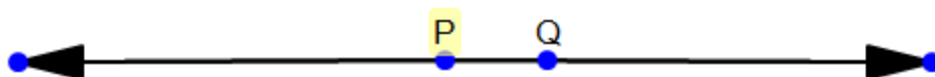


## Distance, Segment, and Rays

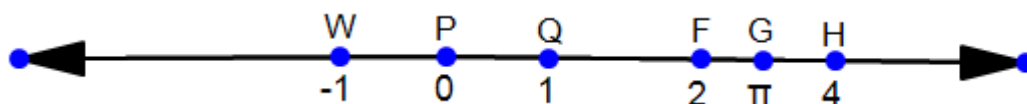
To find the distance between any two points on a line, it is necessary to agree upon a measuring device or ruler. Pick any two points P and Q on a line, with Q to the right of P. Assign the number 0 to P and the number 1 to Q.



The distance from P to Q is 1. Write  $PQ = 1$  or  $QP = 1$ .

$PQ$  or  $QP$  mean “the distance between P and Q”. Using  $PQ$  as a reference; the set of real numbers can now be associated with points on the number line. When this is done, the line is called a **number line**.

Below, point Q corresponds to 1, which is called the **coordinate** of Q. The coordinate of a point on a number line is the number associated with that point.

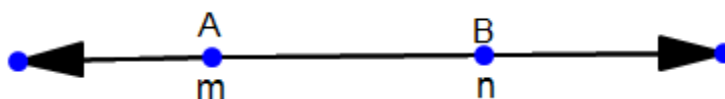


On the line above, the distance between the two points W and F with coordinates  $-1$  and  $2$ , respectively, is equal to the absolute value of the difference of their coordinates.

$$WF = |2 - (-1)| = |3| = 3, \text{ or } WF = |-1 - 2| = |-3| = 3$$

### Definition 1

The distance between any two points A and B with coordinates  $m$  and  $n$  is  $|m - n|$  or  $|n - m|$ .

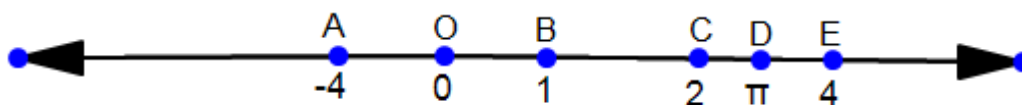


The ideas above are summarized in the Ruler Postulate. A *postulate* is a statement that is accepted without proof.

### Postulate 1: Ruler Postulate

The points on a line can be paired with real numbers so that, given any two points P & Q on the line, P corresponds to zero and Q corresponds to a positive number.

**Example 1:** Find the distance AB, CD and AE using the given number line below.



$$a) AB = |1 - (-4)| = |1 + 4| = |5| = 5$$

$$b) CD = |2 - \pi| = |2 - 3.14| = |-1.14| = 1.14$$

$$c) AE = |-4 - 4| = |-8| = 8$$

In the figure, C is between A and B. There are an infinite number of points between A and B.



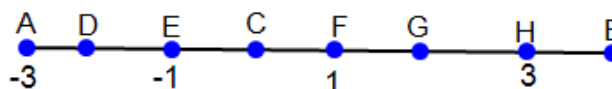
**Definition 2**

A segment is a part of a line that begins at one point and ends at another. The points are called the endpoints of the segment.  
Either AB or BA can be used to name the segment.

**Notes**

- 1) AB is the set of points.
- 2) AB is the distance between points A and B, which is a number.
- 3) The length of AB is AB - the distance between A and B.

In the figure below, C is between A and B.



$$AE = 4 \qquad EH = 5 \qquad AB = 9$$

$$4 + 5 = 9 \text{ Therefore, } AE + EH = AB$$

This suggests the Segment Addition Postulate

**Postulate 2: Segment Addition Postulate**

If C is between A and B, then  $AC + BC = AB$



**Example 2:** Points A, G and R are collinear. Point R is between A and G.  
Draw a diagram. Use the Segment Addition Postulate to write an equation.

Point A can be placed either to the left or to the right of point R.



The equation is  $AR + RG = AG$

or

$GR + RA = GA$ .

**Example 3:** G, R, and A are three collinear points such that A is between G and R.  
 $GA = \frac{3}{5} AR$  and  $GR = 24$ . Find AR and GR.

$$\text{Let } AR = x \Rightarrow GR = \frac{3}{5}x$$

$$GA + AR = GR \quad \text{Write the equation using segment addition postulate}$$

$$\frac{3}{5}x + x = 24 \quad \text{Substitute GA and AR}$$

$$3x + 5x = 120 \quad \text{Multiply the equation by 5}$$

$$8x = 120$$

$$x = 15$$

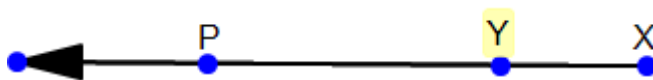
$$AR = x = 15$$

$$GA = \frac{3}{5}x = \frac{3}{5}(15) = 9$$

**Definition 3**

A ray is a part of a line that starts at a point and extends infinitely in one direction. The point is called the endpoint of the ray.

The symbol  $\overrightarrow{XY}$  is used to name the ray.

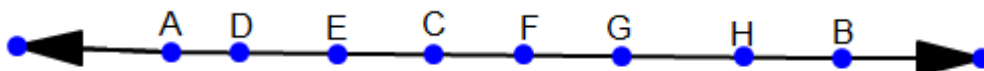


**Note:**  $\overrightarrow{SR}$  and  $\overrightarrow{RT}$  are called opposite rays if S is between R and T.



It is often possible to name a ray in more than one way. However, the first letter always names the endpoint of the ray, and the arrow above the two letters points to the right.

**Example 4:** Refer to the diagram to name:



a) Two segments having E as an endpoint

Possible names for the segments are  $\overline{ED}$ ,  $\overline{EA}$ ,  $\overline{EF}$ ,  $\overline{EB}$ .

b) Three rays having F as an endpoint

Possible names for the ray are  $\overrightarrow{FC}$ ,  $\overrightarrow{FE}$ ,  $\overrightarrow{FD}$ ,  $\overrightarrow{FA}$ .

c) Two points in between points A and G

Possible points are D, E, C, and F