## Congruent Segments

In the figure below, the two segments $\overline{A B}$ and $\overline{C D}$ have the same length. The segment are said to be congruent. The symbol for congruence is $\cong$.The symbol $\nsubseteq$ is read "is not congruent to".


## Definition 1

Congruent ( $\cong$ ) segments are segments that have the same length.
$\overline{A B}$ and $\overline{C D}$ is read "is $\overline{A B}$ congruent to $\overline{C D}$ ".

In the figure, marking sticks (tick marks) are used to indicate that $\overline{\mathrm{AB}}$ and $\overline{\mathrm{CD}}$ are congruent.
You should not assume congruence of segments

unless they are indicated as congruent.
By definition, congruent segments are segments that have the same length. Therefore, the statements $\overline{A B}$ and $\overline{C D}$ and $\mathrm{AB}=\mathrm{CD}$ are equivalent.

## Definition 2

The midpoint, $C$ of $\overline{A B}$ is the point between $A$ and $B$ such that $A C=C B$.


## Definition 3

A segment bisector is a line that divides a segment into two congruent parts. The point where a bisector intersects a segment is the midpoint of the segment.

$\stackrel{\rightharpoonup}{\mathrm{CD}}$ bisect $\overline{\mathrm{PG}}$ at E
$\Rightarrow E$ is the midpoint of $\overline{\mathrm{PG}}$
$\Rightarrow \overline{\mathrm{PE}} \cong \overline{\mathrm{EG}}$

## Postulate 1: Midpoint Postulate

A segment has one and only one midpoint.

Example 1: M is the midpoint of $\overline{A B}$. Find the value of $x$.


M is the midpoint of $\overline{A B}$
$A M=M B$
$5 x+3=33$
$5 x=30$
$x=6$

It is possible to find the coordinate of the midpoint of a segment when the coordinates of the endpoints are given.

The general formula for finding the coordinate of the midpoint of a segment when the coordinates of the endpoints are given is stated below.

## Midpoint Formula

The coordinate, $m$, of the midpoint of a line segment whose endpoints have coordinates $\mathbf{a}$ and $\mathbf{b}$ is $m=\frac{a+b}{2}$.


Example 2: $M$ is the midpoint of $A B$. Find the coordinates of points $A$ and $B$.


$$
\begin{aligned}
& 6=\frac{x-4+x+2}{2} \\
& 6=\frac{2 x-2}{2} \\
& 12=2 x-2 \\
& 14=2 x \\
& 7=x
\end{aligned}
$$

Coordinate of point A: $x-4=7-4=3$

Coordinate of point $\mathrm{B}: ~ \mathrm{x}+2=7+2=9$

## Example 3:



Given: $A B=B C, C$ is the midpoint of $B D$
Prove: $A B=C D$

| Statements | Reasons |
| :--- | :--- |
| 1) $A B=B C$ | 1) Given |
| 2) $B C=C D$ | 2) Definition of a midpoint |
| 3) $A B=C D$ | 3) Transitive |

