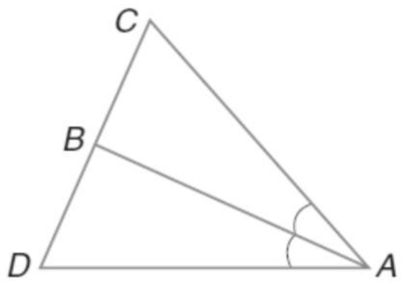
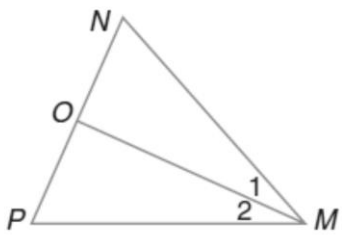
Bisectors of Triangles

An **angle bisector** of a triangle is a segment that separates an angle of the triangle into two congruent angles. One of the endpoints of an angle bisector is a vertex of the triangle, and the other endpoint is on the side opposite that vertex.



$\overline{AB}$ is an angle bisector of $\triangle DAC$.
$\angle DAB \cong \angle CAB$
$m\angle DAB = m\angle CAB$

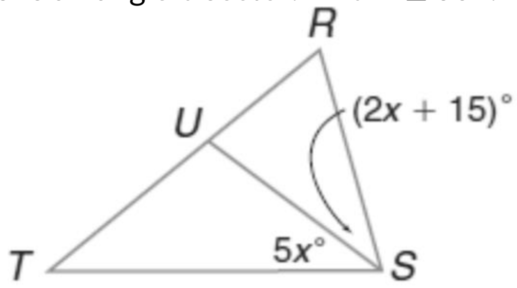
Example 1: In $\sqcup$ MNP, $\overrightarrow{MO}$ bisects $\angle NMP$. If $m\angle 1$ = 33, find $m\angle 2$.



$\overrightarrow{MO}$ bisects $\angle NMP$
$\Rightarrow \angle 1 \cong \angle 2$

$\Rightarrow m\angle 1 = m\angle 2 = 33^0$

Example 2: In □ *RST*, *SU* is an angle bisector. Find *m*∠ *UST*.



SU is an angle bisector.

$\Rightarrow \angle RSU \cong \angle TSU$

$\Rightarrow m\angle RSU = m\angle TSU$

$\Rightarrow 2x + 15 = 5x$

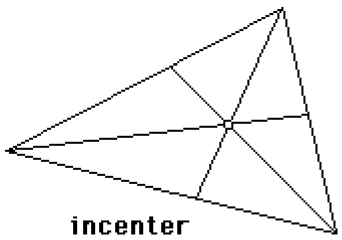
$\Rightarrow 3x = 15$

$\Rightarrow x = 5$

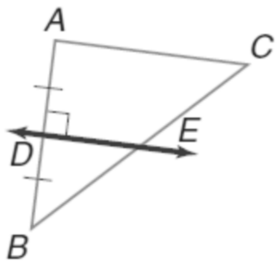
m∠ *UST*=5x=25

<u>Properties of the angle bisector:</u>

- One of the endpoints of an angle bisector is a vertex of the triangle, and the other endpoint is on the side opposite to the vertex.
- Any point on the angle bisector is equidistant from the sides which form the angle.
- The three angle bisectors in a triangle always intersect in one point, and this intersection point always lies in the interior of the triangle.
- The intersection of the three angle bisectors forms the center of the circle in- scribed in the triangle. (The circle which is tangent to all three sides)



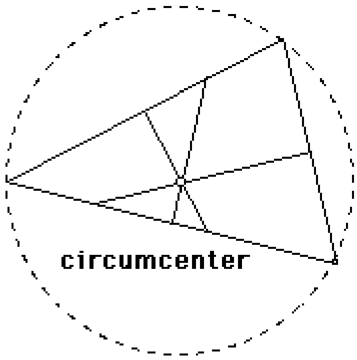
A perpendicular line or segment that bisects a side of a triangle is called the **perpendicular bisector** of that side.



$\overline{DE}$ is the perpendicular bisector of side $\overline{AB}$.

Properties of the perpendicular bisector:

- Any point on the perpendicular bisector of a line segment is equidistant from both endpoints.
- In a triangle the perpendicular bisectors of the three sides always meet in a single point. This point is called the circumcenter.
- If the triangle is acute, the circumcenter lies inside the triangle. If the triangle is obtuse, the circumcenter lies outside the triangle. If the triangle is a right triangle, the circumcenter will coincide with one of the sides.
- The circumcenter is the center of the circumscribed circle. (The circle which passes through all three vertices).



Given a triangle ABC, we can construct four different types of lines with respect to the triangle.
1. The angle bisector bisects an angle to form two congruent angles.
2. The perpendicular bisector: Given a line segment, the perpendicular bisector is the unique perpendicular line passing through the midpoint of the line segment.
3. The median is the line passing through a vertex and the midpoint of the opposite side.
4. The altitude is the line passing through a vertex, perpendicular to the opposite side.

Special Segments in Triangles				
Segment	Altitude	Median	Perpendicular Bisector	Angle Bisector
Type	Line segment	Line segment	Line Line segment	Ray Line segment
Property	From vertex, a line $\perp$ to the opposite side.	From the vertex to the midpoint of the opposite side.	Bisects the side of a triangle.	Bisects the angle of a triangle
Point of intersection	**Orthocenter**	**Centroid**	**Circumcenter**	**In center**