Arithmetic Operations with Rational Numbers

The rules used to add integers and positive rational numbers can be used to add all rational numbers.

Adding Two Signed Numbers

Addition of two numbers with like signs

Step1: Find the sum of the absolute values.

Step2: Use the sign common to both numbers.

Addition of two numbers with unlike signs

Step1: Find the difference of the absolute values.

Step2: Use the sign of the number with greater absolute value.

Example 1: Add

a.
$$-\frac{2}{3} + \left(\frac{-1}{9}\right)$$

$$-\frac{2}{3} + \left(\frac{-1}{9}\right) = -\left(\frac{2}{3} + \frac{1}{9}\right) = -\left(\frac{6}{9} + \frac{1}{9}\right) = -\frac{7}{9}$$

$$1.354 + (-0.765) = +(1.354 - 0.765) = 0.569$$

Subtracting Two Signed Numbers

To subtract two signed number, we add the first number to the add additive inverse of the second number

$$a - b = a + (-b)$$

Note 1: If the sum of two numbers is 0, the numbers are called additive inverses, or opposites.

- -3 is the additive inverse, or opposite, of 3.
- -19.3 the is additive inverse, or opposite, of 19.3.

Additive Inverse Property

For every number a, a + (-a) = 0.

Multiplying Two Signed Numbers

Multiplying two numbers with like signs

Step1: Find the product of the absolute values of the numbers.

Step2: Write the product as a positive number.

$$(+)\bullet(+)=(+)$$
 $(-)\bullet(-)=(+)$

Multiplying two numbers with unlike signs

Step1: Find the product of the absolute values of the numbers.

Step2: Write the product as a negative number.

$$(+)\bullet(-)=(-)$$
 $(-)\bullet(+)=(-)$

Note 2:

The product of an even number of negative factors is positive.

The product of an odd number of negative factors is negative.

Dividing Two Signed Numbers

Dividing two numbers with like signs

Step1: Find the quotient of the absolute values of the numbers.

Step2: Write the quotient as a positive number.

$$(+) \div (+) = (+)$$

$$(-)\div(-)=(+)$$

Dividing two numbers with unlike signs

Step1: Find the quotient of the absolute values of the numbers.

Step2: Write the quotient as a negative number.

$$(+) \div (-) = (-)$$

$$(-) \div (+) = (-)$$

Definition: Two numbers whose product is 1 are multiplicative inverses or reciprocals.

The reciprocal of $\frac{4}{9}$ is $\frac{9}{4}$ because $\frac{4}{9} \cdot \frac{9}{4} = 1$

The reciprocal of -5 is $-\frac{1}{5}$ because $-5(-\frac{1}{5}) = 1$

Multiplicative Inverse Property

For every nonzero number a, there is exactly one number $\frac{1}{a}$, such that $a(\frac{1}{a}) =$

$$\frac{1}{a} (a) = 1.$$

Division Rule

For all numbers a and b, with $b \neq 0$, $a \div b = \frac{a}{b} = a(\frac{1}{b}) = \frac{1}{b}$ (a).