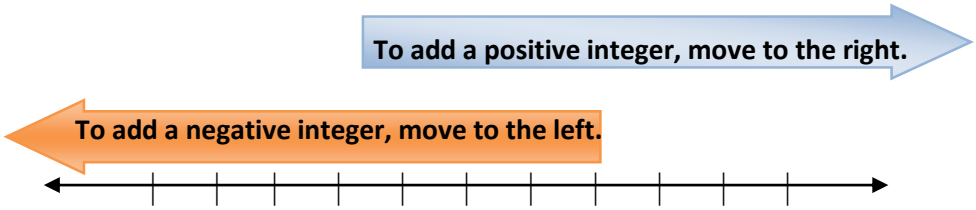
Arithmetic Operations with Integers

Part A: Adding Integers

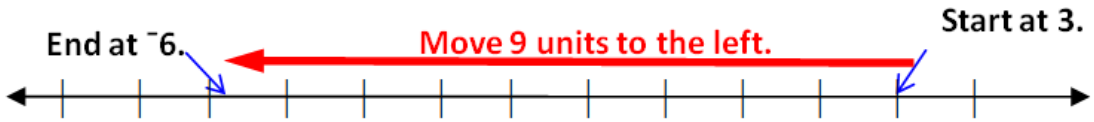
One way to add integers is to use a number line.



Example 1: Use a number line to find the sum.

a) $3 + (^-9)$

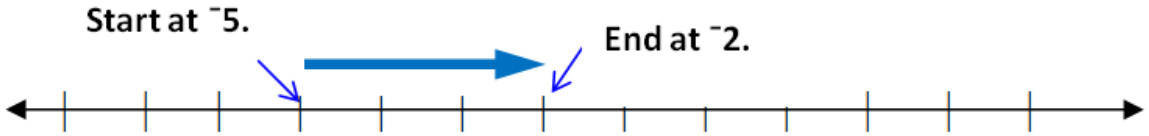
Start at the number 3, we want to add a negative integer so we move to the left 9 units, we will reach -6.



The final position is $^-6$. So, $3 + (^-9) = ^-6$.

b) $^-5 + 3$

Start at the integer -5, we are adding a positive integer so we move to the right 3 units, we reach the integer -2



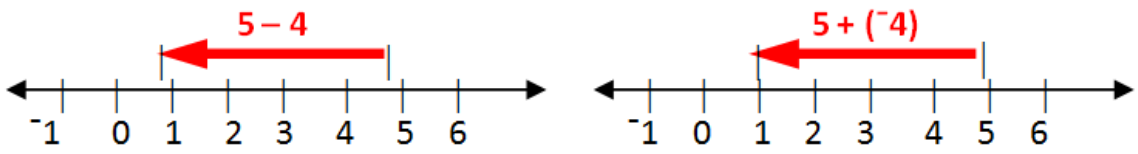
The final position is $^-2$. So, $^-5 + 3 = ^-2$.

Absolute Values: You can use absolute values to find the sum of two or more integers.

Words	Numbers
Adding numbers with the same sign If two numbers have the same sign, add their absolute value and use the sign of the numbers	8 + 12 = 20 $^-6 + (^-4) = ^- 10$
Adding numbers with different signs If two numbers have different signs, find the difference of their absolute values and use the sign of the number with the greater absolute value	$5 + (^-8) = ^-3$ Step1: $^-8$ in absolute value is greater than 5 $^-11 + 13 = 2$
Opposites The sum of a number and its opposite is zero	$7 + (^-7) = 0$

Part B: Subtracting Integers

As you can see from the number lines below, the expression 5 – 4 and 5 + (⁻4) have the same value, 1.



These equivalent expressions suggest the following rule for subtracting integers.

Subtracting Integers	Numbers	Algebra
To subtract a number, add its opposite. Then follow the rules for adding signed numbers	3 – 7 = 3 + (⁻7) = ⁻4	$a - b = a + (^{-}b)$

Part C: Multiplying and Dividing Integers

Use patterns to multiply integers.

1

2

Expression	Product
3(3)	9
3(2)	6
3(1)	3
3(0)	?
3(⁻1)	?
3(⁻2)	?

Expression	Product
2(⁻3)	?
1(⁻3)	?
0(⁻3)	?
⁻1(⁻3)	?
⁻2(⁻3)	?
⁻3(⁻3)	?

You may have recognized patterns in the products of integers. These patterns suggest the following rules.

Words	Numbers
The product of two integers with the same sign is positive.	⁻2(⁻4) = 8 2(4) = 8
The product of two integers with different signs is negative.	⁻2(4) = ⁻8 2(⁻4) = ⁻8
The product of any integer and 0 is 0.	2(0) = 0 ⁻2(0) = 0

Dividing Integers Because 3(⁻4) = ⁻12, you know that ⁻12 ÷ 3 = ⁻4 and ⁻12 ÷ (⁻4) = 3. This relationship between products and quotients suggests that the rules for dividing integers are like the rules for multiplying integers.

Words	Numbers
The quotient of two integers with the same sign is positive.	8 ÷ 4 = 2 ⁻8 ÷ (⁻4) = 2
The quotient of two integers with different signs is negative.	⁻8 ÷ 4 =⁻2 8 ÷ (⁻4) =⁻2
The quotient of 0 and any nonzero integer is 0.	0 ÷ 4 = 0 0 ÷ (⁻4) = 0