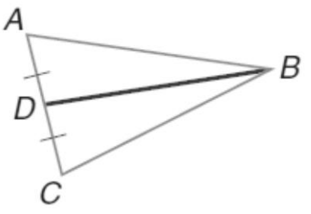
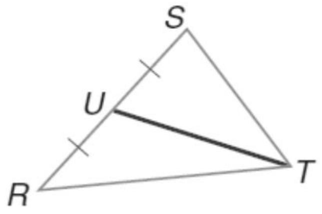
Altitudes and Medians

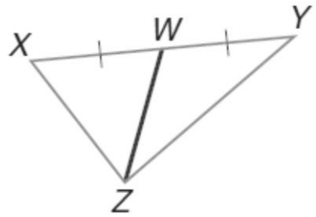
In a triangle, a median is a segment that joins a vertex of the triangle and the midpoint of the side opposite that vertex. In the figures below, a median of each triangle is shown in red.



$\overline{BD}$ *is a median*

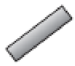


$\overline{TU}$ *is a median*

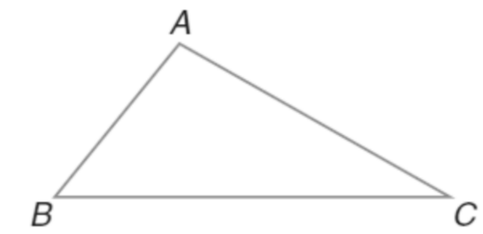


$\overline{WZ}$ *is a median*

Hands On Construction

Materials:  compass  straightedge

Step 1 Draw a triangle like □ *ABC*.

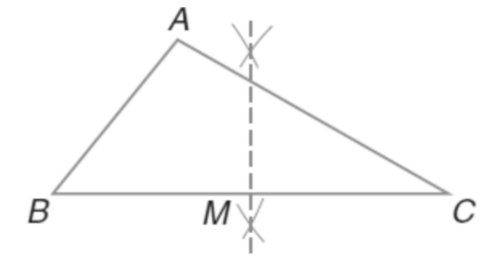


Step 2 The side opposite vertex *A* is *BC*.

Find the midpoint of *BC* by

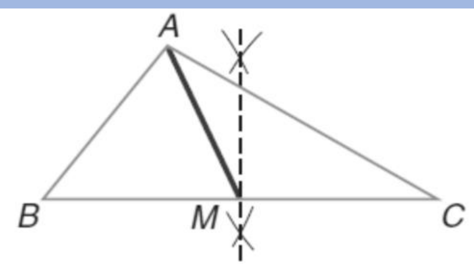
constructing the bisector of *BC*.

Label the midpoint *M*.



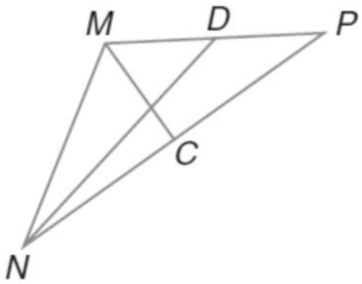
Step 3 Use a straightedge to draw

AM. *AM* is the median of □ *ABC* drawn from vertex *A*.



A triangle has three medians. You can use a compass and a straightedge to construct a median of a triangle.

Example 1: Given $\overline{ND}$ and $\overline{MC}$ are medians



a) If NC=9 cm, find NP

$\overline{MC}$ is a median

$\Rightarrow C$ is the midpoint of $\overline{NP}$

$\Rightarrow \overline{NC} \cong \overline{CP}$

$\Rightarrow NC = CP = \frac{1}{2}NP$

$\Rightarrow 2NC = NP$

$\Rightarrow NP = 2(9) = 18cm$

b) If MP=7 cm, find MD

$\overline{ND}$ is a median

$\Rightarrow D$ is the midpoint of $\overline{MP}$

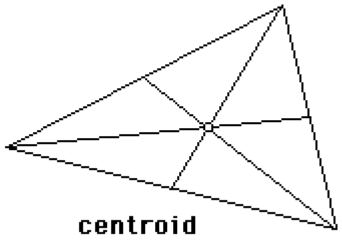
$\Rightarrow \overline{MD} \cong \overline{MP}$

$\Rightarrow MD = DP = \frac{1}{2}MP$

$\Rightarrow MD = \frac{7}{2} = 3.5cm$

<u>Properties of the median:</u>

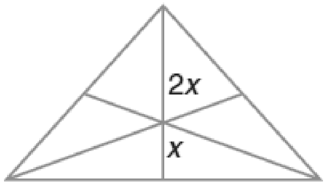
- The medians of a triangle always intersect in one point called the centroid.
- The centroid always lies inside the triangle.
- The centroid divides the median into two segments. The lengths of these two segments always have a constant ratio.
- The length of the segment from the vertex to the centroid is twice the length of the segment from the centroid to the midpoint.



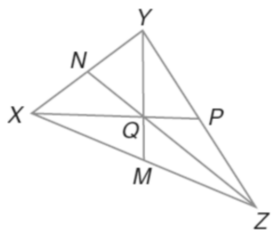
There is a special relationship between the length of the segment from the vertex to the centroid and the length of the segment from the centroid to the midpoint.

Theorem 1

The length of the segment from the vertex to the centroid is twice the length of the segment from the centroid to the midpoint.



Example 2: In □ *XYZ*, *XP*, *ZN*, and *YM* are medians.



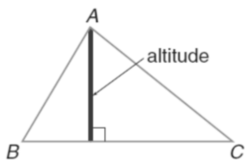
a) Find *ZQ* if *QN* = 5.

Since *QN* = 5, *ZQ* = 2 × 5 = 10.

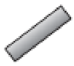
b) If *XP* = 10, what is *QP*?

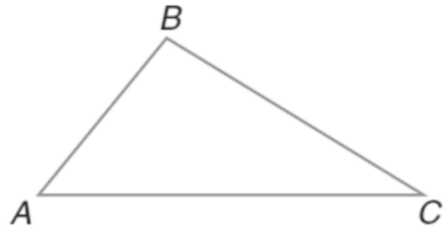
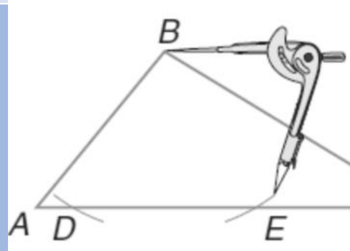
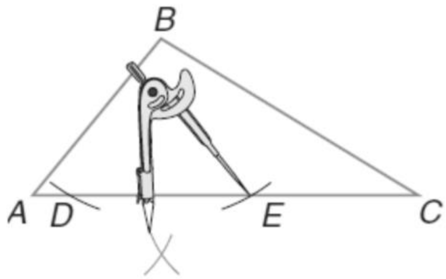
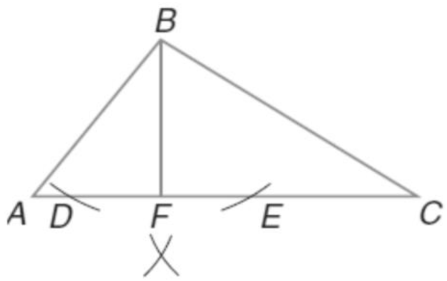
Since *XP* =10.5, *QP* $= \frac{1}{3} \times 10 = \frac{10}{3} = 3.3$

In geometry, an altitude of a triangle is a perpendicular segment with one endpoint at a vertex and the other endpoint on the side opposite that vertex. The altitude *AD* is perpendicular to side *BC*.



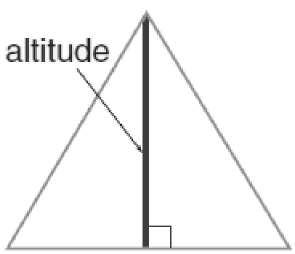
Hands On Construction

Materials:  compass  straightedge

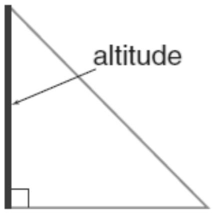
Step 1 Draw a triangle like □ *ABC*.	
Step 2 Place the compass point at *B* and draw an arc that intersects side *AC* in two points. Label the points of intersection *D* and *E*.	
Step 3 Place the compass point at *D* and draw an arc below *AC*. Using the same compass setting, place the compass point at *E* and draw an arc to intersect the one drawn.	
Step 4 Use a straightedge to align the vertex *B* and the point where the two arcs intersect. Draw a segment from vertex *B* to side *AC*. Label the point of intersection *F*.	

An altitude of a triangle may not always lie inside the triangle.

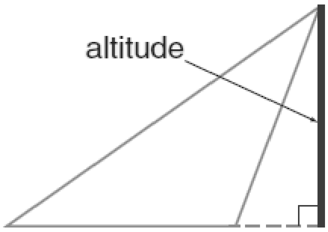
If the triangle is an acute triangle then the altitude is inside the triangle



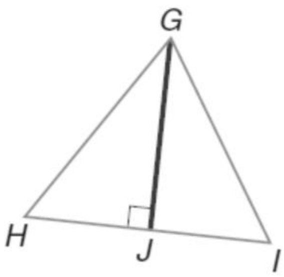
If the triangle is right then the altitude is the side of the triangle



If the triangle is obtuse then the altitude is outside the triangle



Example 3: $\overline{GJ}$ is an altitude, if $m\angle GIJ = 48^0$, find $m\angle IGJ$



$\overline{GJ}$ is an altitude

$\Rightarrow \overline{GJ} \perp \overline{IH}$

$\Rightarrow \angle GJI$ is a right angle

In $\triangle GJI$, we have:

$m\angle GJI + m\angle GIJ + m\angle JGI = 180^0$

$\Rightarrow 90^0 + 48^0 + m\angle JGI = 180^0$

$\Rightarrow m\angle JGI = 180^0 - 138^0 = 42^0$

Properties of the altitude:

- The altitudes of a triangle always intersect in one point called the orthocenter.
- If the triangle is acute, the intersection point lies inside the triangle.
- If the triangle is obtuse, the intersection point lies outside the triangle.
- If the triangle is a right triangle, the intersection point will coincide with the vertex which represents the right angle.