

Algebraic Properties

Part A: Commutative and Associative Properties

In English, commute means to change locations, and associate means to group together. These words have similar meaning in mathematics. Commutative properties let you change the positions of numbers in a sum or product. Associative properties let you group numbers in a sum or product together.

Commutative and Associative Properties	
<p>Commutative Property of Addition</p> <p>Words: In a sum, you can add the numbers in any order.</p> <p>Numbers: $4 + (-7) = -7 + 4$</p> <p>Algebra: $a + b = b + a$</p>	<p>Commutative Property of Multiplication</p> <p>Words: In a product, you can multiply the numbers in any order.</p> <p>Numbers: $8(-5) = -5(8)$</p> <p>Algebra: $ab = ba$</p>
<p>Associative Property of Addition</p> <p>Words: Changing the grouping of the numbers in a sum does not change the sum.</p> <p>Numbers: $(9 + 6) + 2 = 9 + (6+2)$</p> <p>Algebra: $(a + b) + c = a + (b + c)$</p>	<p>Associative Property of Multiplication</p> <p>Words: Changing the grouping of the numbers in a product does not change the product.</p> <p>Numbers: $(3 \cdot 10) \cdot 4 = 3 \cdot (10 \cdot 4)$</p> <p>Algebra: $(ab)c = a(bc)$</p>

Example 1: You buy a portable CD player for \$48, rechargeable batteries with charger for \$25, and a CD case for \$12. Find the total cost.

The total cost is the sum of the three prices. Use properties of addition to group together prices that are easy to add mentally.

$$\begin{aligned}
 48 + 25 + 12 &= (48 + 25) + 12 \\
 &= (25 + 48) + 12 \\
 &= 25 + (48 + 12) \\
 &= 25 + 60 \\
 &= 85
 \end{aligned}$$

Use order of operations.
 Commutative property of addition
 Associative property of addition
 Add 48 and 12.
 Add 25 and 60

The total cost is \$85.

Part B: Identity Properties

Identity Properties: When 0 is added to any number, or when any number is multiplied by 1, the result is identical to the original number. These properties of 0 and 1 are called identity properties, and the numbers 0 and 1 are called identities.

Identity Properties	
Identity Property of Addition Words: The sum of a number and the additive identity , 0, is the number. Numbers: $-6 + 0 = -6$ Algebra: $a + 0 = a$	Identity Property of Multiplication Words: The product of a number and the multiplicative identity , 1, is the number. Numbers: $4 \cdot 1 = 4$ Algebra: $a \cdot 1 = a$

Part C: The Distributive Property

The Distributive Property

Algebra

$$\begin{aligned} a(b + c) &= ab + ac \\ (b + c)a &= ba + ca \\ a(b - c) &= ab - ac \\ (b - c)a &= ba - ca \end{aligned}$$

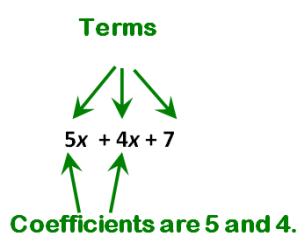
Numbers

$$\begin{aligned} 4(6 + 3) &= 4(6) + 4(3) \\ (6 + 3)4 &= 6(4) + 3(4) \\ 5(7 - 2) &= 5(7) - 5(2) \\ (7 - 2)5 &= 7(5) - 2(5) \end{aligned}$$

Note: The expressions $2(90 + 60)$ and $2(90) + 2(60)$ are called **equivalent numerical expressions** because they have the same value. The statement $2(90 + 60) = 2(90) + 2(60)$ illustrates the distributive property for evaluating the product of a number and a sum or difference.

Two variable expressions that have the same value for all values of the variable(s) are called **equivalent variable expressions**. You can use the distributive property to write equivalent variable expressions.

The parts of an expression that are added together are called **terms**. In the expression below, the terms are $5x$, $4x$, and 7 . The **coefficient** of a term with a variable is the number part of the term.



A **constant term**, such as 7, has a number but no variable.

Like terms are terms that have identical variable parts. In the expression above, $5x$ and $4x$ are like terms. Two or more constant terms are also considered like terms.

Identify the terms, like terms, coefficients, and constant terms of the expression $y + 8 - 5y - 3$.

Solution

Write the expression as a sum: $y + 8 + (-5y) + (-3)$.

Note: $y = 1y$, the coefficient of y is 1.

Terms: $y, 8, -5y, -3$

Like terms: y and $-5y$; 8 and -3

Coefficients: $1, -5$

Constant terms: $8, -3$

You can use the distributive property to write an expression such as $7x + 4x$ as a single term:

$$7x + 4x = (7 + 4)x = 11x$$

The like terms $7x$ and $4x$ have been combined, and the expression $7x + 4x$ has been simplified. A variable expression is simplified if it contains no grouping symbols and all like terms are combined.