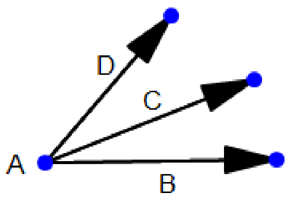
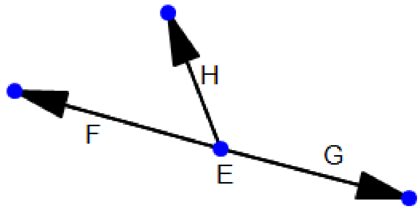
Adjacent Angles and Angle Bisector

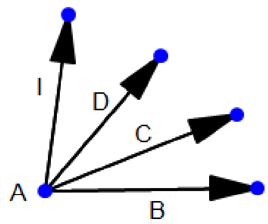
The figures below show examples of **adjacent** and **not adjacent** angles respectively.
Adjacent angles

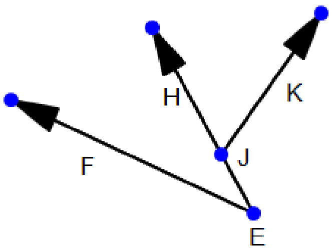


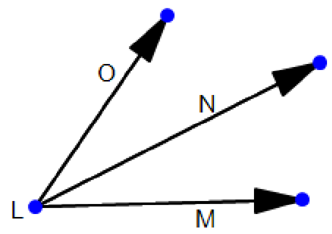


∠BAC and ∠DAC are adjacent angles. ∠FEH and ∠HEG are adjacent angles.

Not adjacent angles







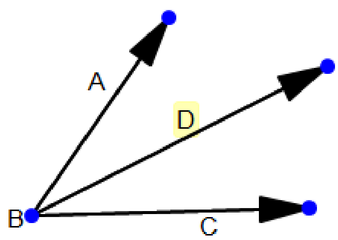
∠BAC and ∠DAI are non adjacent angles.

∠FEH and ∠HJK are non adjacent angles.

∠MLN and ∠MLO are non adjacent angles.

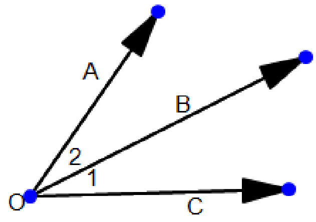
Definition 1
Adjacent angles are two angles in a plane that have a common vertex and a common side but no common interior points.

The figure at the right shows adjacent angles, ∠ABD and ∠DBC. The common side of the angles is $\overrightarrow{BD}$. The rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are called the outer rays of the angles.
Notice that $m\angle ABC = 40 + 20$, or 60.



This suggests the following postulate.

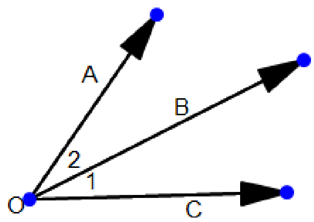
Example 1: Given $m\angle AOC = 110$, $m\angle 2 = 50$. Find $m\angle 1$.



Use the angle addition postulate to write an equation

$$m\angle 1 + m\angle 2 = m\angle AOC$$
$$m\angle 1 + 50 = 110$$
$$m\angle 1 = 110 - 50$$
$$m\angle 1 = 60$$

Example 2: Given $m\angle AOC = 140$, $m\angle 1 = \frac{2}{3} m\angle 2$. Find $m\angle 1$ *and* $m\angle 2$



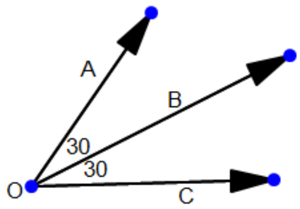
Use the angle addition postulate to write an equation.

$$\text{Let } m\angle 2 = x \Rightarrow m\angle 1 = \frac{2}{3}x$$
$$m\angle 1 + m\angle 2 = m\angle AOC$$
$$\frac{2}{3}x + x = 140$$
$$\frac{5}{3}x = 140$$
$$x = 140 \bullet \frac{3}{5}$$
$$x = 84$$

$$m\angle 1 = \frac{2}{3}x = \frac{2}{3} \bullet 84 = 56$$

$$m\angle 2 = x = 84$$

Consider the figure given at the right.

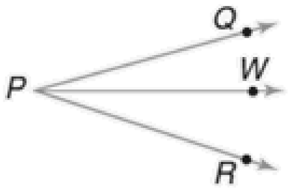


$m\angle AOB = m\angle BOC = 30 \Rightarrow \angle AOB \cong \angle BOC$

$\overrightarrow{OC}$ bisects $\angle AOC$

Definition 2: Angle Bisector

The bisector of an angle is the ray with its endpoint at the vertex of the angle, extending into the interior of the angle. The bisector separates the angle into two angles of equal measure



$\overrightarrow{PW}$ **is the bisector of** $\angle RPQ$

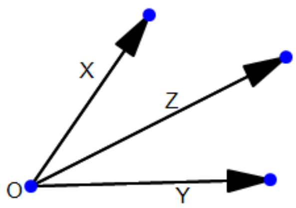
$\angle RPW \cong \angle WPQ$

$m\angle RPW = m\angle WPQ$

Postulate Angle Bisector Postulate

Every angle, except a straight angle, has exactly one bisector.

Example 3: Given that $\overrightarrow{OZ}$ bisects $\angle XOY$,
$m\angle XOZ = 5x + 4, m\angle YOZ = 7x - 10$. Find $m\angle XOZ$



$m\angle XOZ = m\angle YOZ$ Definition of angle bisector.

$5x + 4 = 7x - 10$

$10 + 4 = 7x - 5x$

$14 = 2x$

$7 = x$

$m\angle XOZ = 5x + 4 = 5 \bullet 7 + 4 = 39$