AAS, HA, LL and HL Theorems

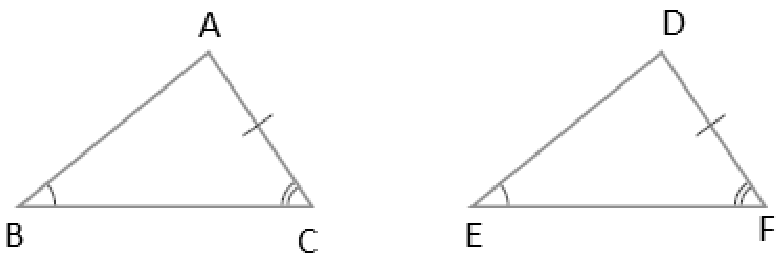
If two angles of one triangle are congruent to two angles in another triangle then the third angle of the first triangle is congruent to the third angle of the second triangle.

So when we need to prove that two triangles are congruent and we have two pairs of congruent angles and a pair of congruent sides then we can use the triangle angle sum theorem to prove the third pair of angles is congruent; thus using the ASA Postulate the two triangles are congruent

Since the above argument is always true then the following theorem can be used to prove that two triangles are congruent.

Theorem 1: Angle-Angle-Side Congruency Theorem: (AAS theorem)

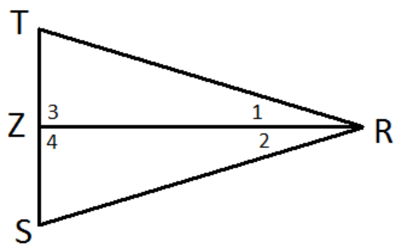
If two angles and a non included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle, the two triangles are congruent



$$\textbf{If}\ \left.\begin{array}{l} \angle ABC \cong \angle DEF \\ \angle ACB \cong \angle DFE \\ \overline{AC} \cong \overline{DF} \end{array}\right\} \Rightarrow \triangle BAC \cong \triangle EDF$$

Example 1:

Given: $\angle 1 \cong \angle 2$
$\angle S \cong \angle T$



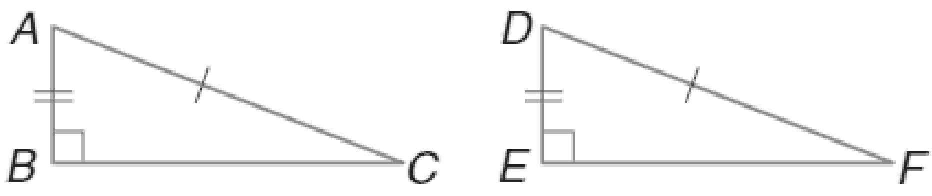
Prove: $\Delta RZS \cong \Delta RZT$

Statements	Reasons
1) $\angle 1 \cong \angle 2$	**1)** Given
2) $\angle S \cong \angle T$	**2)** Given
3) $\overline{RZ} \cong \overline{RZ}$	**3)** Reflexive
4) $\Delta RZS \cong \Delta RZT$	**4)** AAS theorem

There will be a special set of theorems and a postulate to prove that two right triangles are congruent. To use HL postulate, HA theorem, LL theorem or LA theorem the triangles must be right.

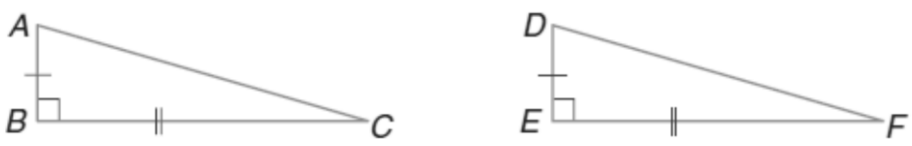
Postulate: HL Postulate

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.



If $\left.\begin{array}{l}\angle ABC \cong \angle DEF \\ \angle ACB \cong \angle DFE \\ \overline{AC} \cong \overline{DF}\end{array}\right\} \Rightarrow \triangle BAC \cong \triangle EDF$

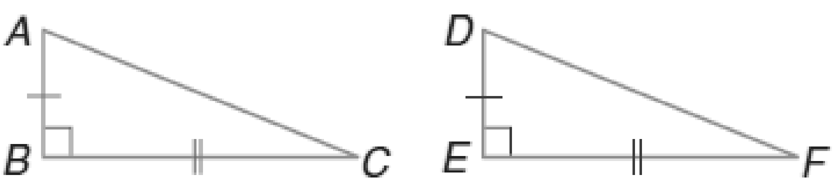
In triangles *ABC* and *DEF*, the two right angles are congruent. Also, the corresponding legs are congruent. So, the triangles are congruent by SAS.



Since right triangles are special cases of triangles, the SAS test for congruence can be used to establish the following theorem.

Theorem 2: LL Theorem

If two legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.



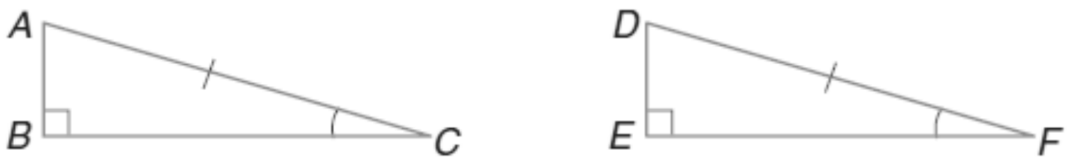
If $\left.\begin{array}{l}\overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF}\end{array}\right\} \Rightarrow \triangle BAC \cong \triangle EDF$

Suppose the hypotenuse and an acute angle of the triangle on the left are congruent to the hypotenuse and acute angle of the triangle on the right.

Since the right angles in each triangle are congruent, the triangles are congruent by AAS.

Theorem 3: HA Theorem

If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and corresponding angle of another right triangle, then the triangles are congruent.

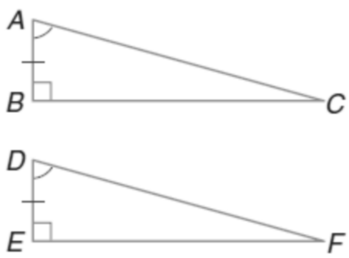


$$\text{If } \left.\begin{array}{l} \angle ACB \cong \angle DFE \\ \overline{AC} \cong \overline{DF} \end{array}\right\} \Rightarrow \triangle BAC \cong \triangle EDF$$

Suppose a leg and an acute angle of one triangle are congruent to the corresponding leg and acute angle of another triangle.

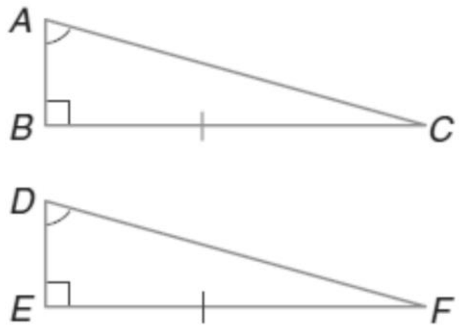
Case 1

The leg is included between the acute angle and the right angle.



Case 2

The leg is not included between the acute angle and the right angle.

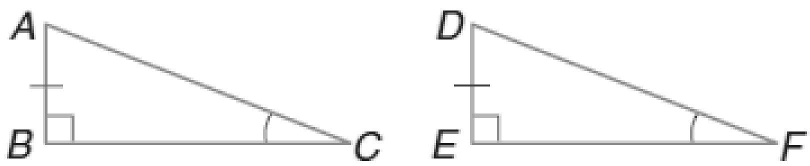


The right angles in each triangle are congruent.

In Case 1, the triangles are congruent by ASA. In Case 2, the triangles are congruent by AAS. This leads to Theorem

Theorem 4: LA Theorem

If one leg and an acute angle of a right triangle are congruent to the corresponding leg and angle of another right triangle, then the triangles are congruent.



$$\text{If } \left.\begin{array}{l} \angle ACB \cong \angle DFE \\ \overline{AB} \cong \overline{DE} \end{array}\right\} \Rightarrow \triangle BAC \cong \triangle EDF$$

The following postulate describes the congruence of two right triangles when the hypotenuse and a leg of the two triangles are congruent.