Volume: The Shell Method

Cylindrical Shell Method

If the cross sections of the solid are taken parallel to the axis of Revolution, then the cylindrical shell method will be used to find the volume of the solid. If the cylindrical shell has radius *r* and height *h*, then its volume would be $2\pi rh$ times its thickness. Think of the first part of this product, $(2\pi rh)$, as the area of the rectangle formed by cutting the shell perpendicular to its radius and laying it out flat. If the axis of revolution is vertical, then the radius and height should be expressed in terms of *x*. If, however, the axis of revolution is horizontal, then the radius and height should be expressed in terms of *y*.

The volume (*V*) of a solid generated by revolving the region bounded by $y = f(x)$ and the *x*-axis on the interval $[a,b]$, where $f(x) \geq 0$, about the *y*-axis is $V = \int_a^b 2\pi x f(x)dx$

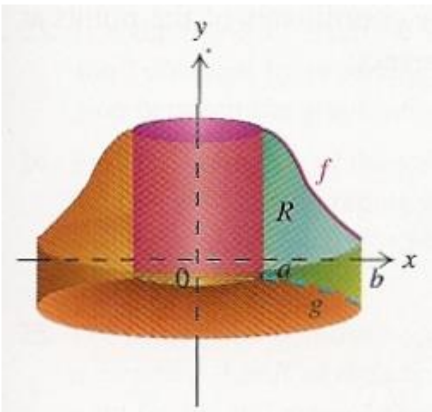
If the region bounded by $x = f(y)$ and the *y*-axis on the interval $[a,b]$, where $f(y) \geq 0$, is revolved about the *x*-axis, then its volume (*V*) is $V = \int_a^b 2\pi y f(y)dy$

Note that the *x* and *y* in the integrands represent the radii of the cylindrical shells or the distance between the cylindrical shell and the axis of revolution. The $f(x)$ and $f(y)$ factors represent the heights of the cylindrical shells.

Finally, let f and g be continuous on $[a,b]$, with $a \geq 0$, and suppose that

$g(x) \leq f(x)$ for $a \leq x \leq b$

then let R be the region between the graphs of f and g on $[a,b]$. The volume V of the solid obtained by revolving R about the y axis is given by $V = \int_a^b 2\pi x\left[f(x) - g(x)\right]dx$



we will use $V=\int_a^b 2\pi x\left[f(x)-g(x)\right]dx$ to find the volume of a solid rather like a single-scoop ice cream cone

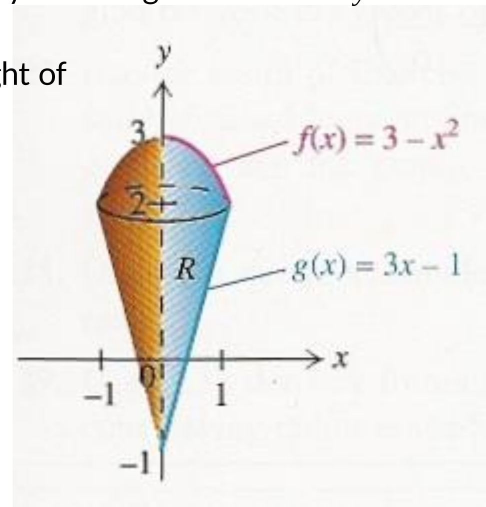
Example 1: let $f(x)=3-x^2$ and $g(x)=3x-1$, and let R be the region between the graphs of f and g on $[0,1]$. Find the volume V of the solid generated by revolving R about the y axis.

since $3-x^2\geq 3x-1$ for $0\leq x\leq 1$, it follows that the height of the solid at any x in $[0,1]$ is $(3-x^2)-(3x-1)$.

$$V=\int_0^1 2\pi x\left[\left(3-x^2\right)-\left(3x-1\right)\right]dx$$

$$=2\pi\int_0^1\left(-x^3-3x^2+4x\right)dx=2\pi\left(-\frac{1}{4}x^4-x^3+2x^2\right)\Bigg|_0^1$$

$$=\frac{3\pi}{2}\text{ cubic units}$$



The volumes of certain solids of revolution can be evaluated either by the washer method or by the shell method. As you would expect, the result is the same, whichever method is used. We support this claim with the following example.