# Unit Vector and Direction Angles

**Unit Vectors:**

Definition 1: A unit vector has length (or magnitude) 1. Unit vectors are often denoted by u. Given a vector v, the unit vector in the direction of v is given by:

$$u = \frac{1}{\|v\|} v \quad (\mathbf{v} \neq \mathbf{0})$$

Note that u is a scalar multiple of v. The vector u has a magnitude of 1 and the same direction as v. The vector u is called a unit vector in the direction of v.

Example 1: Find the unit vector in the direction of **v** = ⟨3, -4⟩.

$$\|v\| = \sqrt{3^2 + (-4)^2} = 5$$

The unit vector in the direction of v is u:

$$u = \frac{1}{5}\langle 3, -4\rangle = \left\langle \frac{3}{5}, \frac{-4}{5} \right\rangle$$

**Standard Unit Vectors:** $i = \langle 1, 0\rangle$ and $j = \langle 0, 1\rangle$

Definition 2: For $\mathbf{a} = \langle a_1, a_2\rangle$, it could be written using unit vectors as $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$. This form is called a **linear combination** of the unit vectors **i** and **j**.
For example $v = \langle -5, 12\rangle$ can be written as $v = -5\mathrm{i} + 12\mathrm{j}$

Remarks:

1) If $\|v\| = 1$, $v$ is a unit vector. (A vector of magnitude one is called a **unit vector**). There are two special unit vectors used in the *xy*-plane: $\mathbf{i} = \langle 1, 0\rangle$ and $\mathbf{j} = \langle 0, 1\rangle$.
2) If $\|v\| = 0 \Leftrightarrow v$ is a zero vector. (A vector of magnitude zero is called the **zero vector**. By definition, the zero vector is $\mathbf{0} = \langle 0, 0\rangle$).

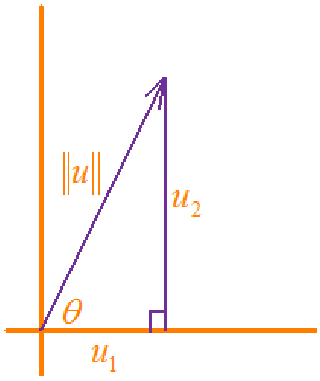
## Formulas for Horizontal and Vertical Components of a Vector u

Definition 3: Let $\theta$ be an angle in standard position, measured from the positive x-axis to the vector $u = \langle u_1, u_2 \rangle = u_1\mathrm{i} + u_2\mathrm{j}$. Then the horizontal and vertical components, $u_1$ and $u_2$ respectively, can be found as follows.

$$u_1 = \|\mathrm{u}\|\cos\theta \Rightarrow \cos\theta = \frac{u_1}{\|\mathrm{u}\|}$$

$$u_2 = \|\mathrm{u}\|\sin\theta \Rightarrow \sin\theta = \frac{u_2}{\|\mathrm{u}\|}$$

$$u = \langle \|u\|\cos\theta, \|u\|\sin\theta \rangle$$




Definition 4: If u is the unit vector such that $\theta$ is the angle (measured counterclockwise)from the positive x-axis to u, the terminal point of u lies on the unit circle and you have
$u = \langle x, y \rangle = \langle \cos\theta, \sin\theta \rangle = (\cos\theta)i + (\sin\theta)j$
The angle $\theta$ is called the direction angle of the vector u.

Suppose that u is a unit vector with direction angle $\theta$. If v=ai+bj is any vector that makes an angle $\theta$ with the positive x-axis and it has the same direction as u, you can write:

$$v = \|v\|\langle \cos\theta, \sin\theta \rangle$$

$$v = \|v\|(\cos\theta)i + \|v\|(\sin\theta)j$$

Because $v = ai + bj = \|v\|(\cos\theta)i + \|v\|(\sin\theta)j$, it follows that the direction angle $\theta$ for v is determined by:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\|v\|\sin\theta}{\|v\|\cos\theta} = \frac{b}{a}$$