

## Trigonometric Limits

The limits of many algebraic functions can be evaluated by direct substitution. The six trigonometric functions also exhibit this desirable quality.

### Theorem 1: Limits of Trigonometric Functions

Let  $c$  be a real number in the domain of the given trigonometric function.

$$1) \lim_{x \rightarrow c} \sin x = \sin c$$

$$2) \lim_{x \rightarrow c} \cos x = \cos c$$

$$3) \lim_{x \rightarrow c} \tan x = \tan c$$

$$4) \lim_{x \rightarrow c} \cot x = \cot c$$

$$5) \lim_{x \rightarrow c} \sec x = \sec c$$

$$6) \lim_{x \rightarrow c} \csc x = \csc c$$

### Example 1: Find:

$$1) \lim_{x \rightarrow 0} \tan x$$

$$\lim_{x \rightarrow 0} \tan x = \tan 0 = 0$$

$$2) \lim_{x \rightarrow \pi} x \cos x$$

$$\lim_{x \rightarrow \pi} x \cos x = \pi \cos \pi = \pi(-1) = -\pi$$

$$3) \lim_{x \rightarrow \frac{\pi}{2}} \sin^2 x$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \sin^2 x = \sin^2 \frac{\pi}{2} = \left( \sin \frac{\pi}{2} \right)^2 = (1)^2 = 1$$

### Theorem 2: Three Special Trigonometric Limits

$$1. \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$2. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$3. \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$