Trigonometric Integrals

Strategy for Trigonometric Integrals

For $\int \sin^m x \cos^n x dx \quad (m, n \geq 0)$

Case 1: If ***m*** (the power of sine) is **even** and ***n*** (the power of cosine) is **odd**:
Let $u = \sin x$, save one factor of $\cos x$ for du, and use $\cos^2 x = 1 - \sin^2 x$ to express the remaining factors in terms of sine.

Example 1: Find $\int \sin^4 x \cos^3 x dx$

Let, $u = \sin x$ and $du = \cos x dx$

$$\int \sin^4 x \cos^3 x dx$$
$$= \int \sin^4 x \cos^2 x \cos x dx$$
$$= \int \sin^4 x \left(1 - \sin^2 x\right) \cos x dx$$
$$= \int u^4 \left(1 - u^2\right) du$$
$$= \frac{u^5}{5} - \frac{u^7}{7}$$
$$= \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$$

Case 2: If ***m*** (the power of sine) is **odd** and ***n*** (the power of cosine) is **even**:
Let $u = \cos x$, save one factor of $\sin x$ for du, and use $\sin^2 x = 1 - \cos^2 x$ to express the remaining factors in terms of cosine.

Example 2: Find $\int \sin^5 x \cos^2 x dx$

Let, $u = \cos x$ and $du = -\sin x dx$

$$\int \sin^5 x \cos^2 x dx$$
$$= \int \sin^4 x \cos^2 x \sin x dx$$
$$= \int \left(1 - \cos^2 x\right)^2 \cos^2 x \sin x dx$$
$$= \int -\left(1 - u^2\right)^2 u^2 du$$
$$= \int -\left(u^2 - 2u^3 + u^6\right) du$$
$$= -\frac{u^3}{3} + 2\frac{u^4}{4} - \frac{u^7}{7}$$
$$= -\frac{\cos^3 x}{3} + 2\frac{\cos^4 x}{4} - \frac{\cos^7 x}{7} + C$$

Case 3: If ***m*** (the power of sine) is **odd** and ***n*** (the power of cosine) is **odd**:
Use either one of the above two methods.

Case4: If ***m*** (the power of sine) is **even** and ***n*** (the power of cosine) is **even**:
You have to use the identities: $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ and $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Note:
1. If either or both powers of sine and cosine are odd, you may need to use one of the following identities: $\cos^2 x = 1 - \sin^2 x$ and $\sin^2 x = 1 - \cos^2 x$ (both of these are from $\cos^2 x + \sin^2 x = 1$).

2. If both powers of sine and cosine are even, you must use the identities: $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ and $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

3. Always keep in mind the identities:
$(a + b)(a - b) = a^2 - b^2$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$

For $\int \tan^m x \sec^n x dx \ (m, n \geq 0)$

Case 1: If ***n*** (the power of secant) is **even**:
Let $u = \tan x$, save one factor of $\sec^2 x$ for du and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factor of secant in terms of tangent.

Case 2: If ***m*** (the power of tangent) is **odd**:
Let $u = \sec x$, save one factor of $\tan x \sec x$ for du and use $\sec^2 x - 1 = \tan^2 x$ to express the remaining factor of tangent in terms of secant.

Summary:

Power of tangent	Power of secant	How to solve it?
Odd	Odd	Use case 2
Odd	Even	Use case 1 or 2
Even	Odd	Integration by Parts
Even	Even	Use case 1