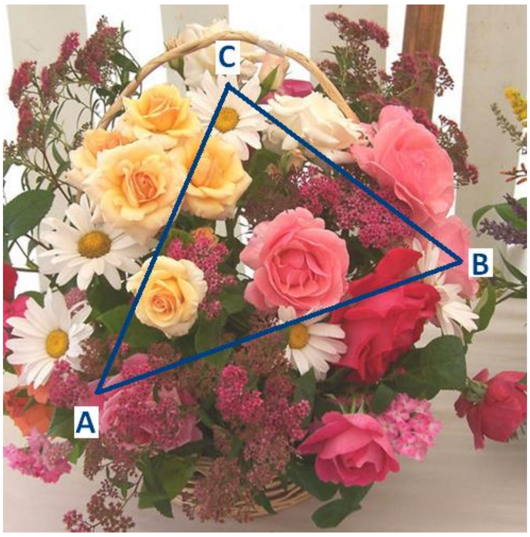
Triangle Inequality

Florists often use triangles as guides in their flower arrangements. There are special relationships between the side measures and angle measures of each triangle. You will discover these relationships in the following activity.

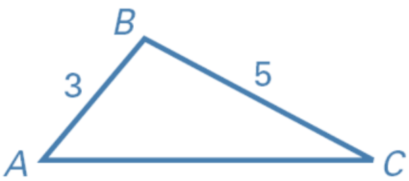


Suppose in triangle *ABC*, the inequality *AC* > *BC* holds true. Is there a similar relationship between the angles ∠*B* and ∠*A*, which are across from those sides?

The observations we made in the previous activity suggest the following theorems.

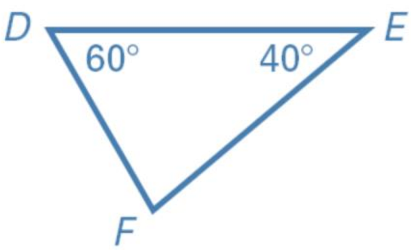
Theorem 1: If one side of a triangle is longer than another side, then the angle opposite to the longer side is larger than the angle opposite to the shorter side.

$$3 < 5$$
$$\Rightarrow AB < BC$$
$$\Rightarrow m\angle BCA < m\angle BAC$$



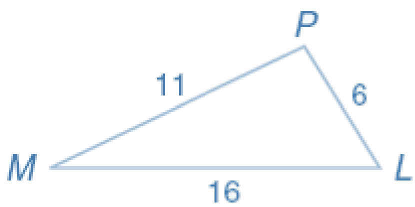
Theorem 2: If one angle of a triangle is larger than another angle, then the side opposite to the larger angle is longer than the side opposite to the smaller angle.

$$40^0 < 60^0$$
$$\Rightarrow m\angle FED < m\angle FDE$$
$$\Rightarrow DF < FE$$



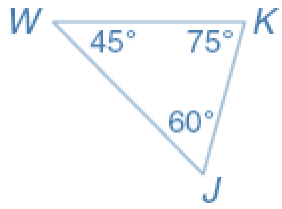
Theorem 3: If the measures of three sides of a triangle are unequal, then the measures of the angles opposite to those sides are unequal in the same order.

$$PL < MP < LM$$
$$\Rightarrow m\angle M < m\angle L < m\angle P$$

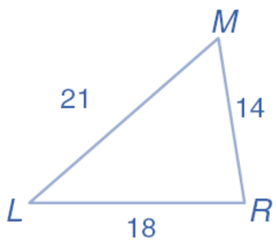


Theorem 4: If the measures of three angles of a triangle are unequal, then the measures of the sides opposite to those angles are unequal in the same order.

$$m\angle W < m\angle J < m\angle K$$
$$\Rightarrow \; JK < KW < WJ$$



Example 1: **In □ *LMR*, list the angles in order from least to greatest measure.**

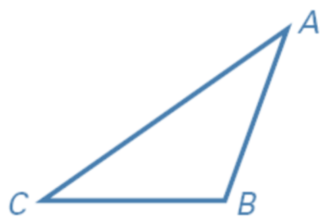


First, write the segment measures in order from least to greatest.
MR < *RL* < *LM*
Use **theorem 3** to write the measures of the angles opposite those sides in the same order.
The angles in order from least to greatest measure are $\angle L$, $\angle M$, and $\angle R$.
$\Rightarrow m\angle L < m\angle M < m\angle R$

Think! Can you always make a triangle with any three line segments? For example, three segments of lengths 1 centimeter, 1.5 centimeters, and 3 centimeters are given.

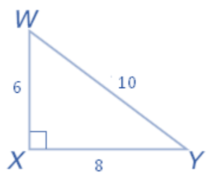
Theorem 5: Triangle Inequality: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$AC + AB > BC$$
$$AB + BC > AC$$
$$AC + BC > AB$$

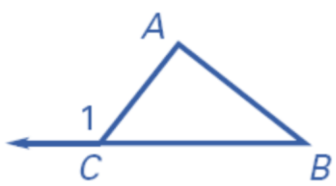


According to the Triangle Inequality Theorem, it is not possible to make a triangle with the three segments. Why? The sum of any two sides of a triangle has to be greater than the third side.

Theorem 6: In a right triangle, the hypotenuse is the side with the greatest measure.



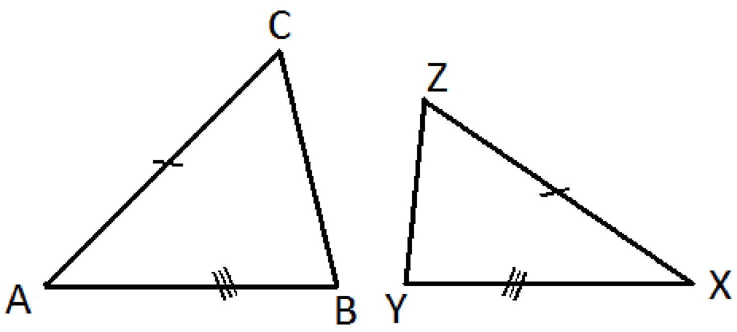
Theorem 7: **Exterior Angle Inequality:** The measure of an exterior angle of a triangle is greater than the measure of either of the two nonadjacent interior angles.



Inequalities Involving Two Triangles

Theorem 8: **SAS Inequality:** If two sides of one triangle are congruent to two sides of another triangle, and the included angle in one triangle is greater than the included angle in the other, then the third side of the first triangle is longer than the third side in the second triangle.

Theorem 9: **SSS Inequality:** If two sides on one triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle.



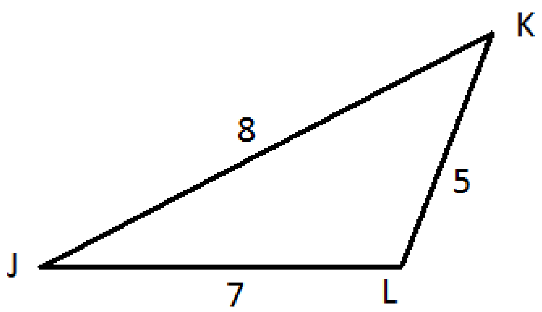
SSS inequality theorem:

$\overline{AB} \cong \overline{XY}$ and $\overline{AC} \cong \overline{XZ}$

$BC > YZ$, then $m\angle A > m\angle X$

Example: Given: $\triangle JKL$

Prove: $m\angle K < m\angle L$



Assume that $m\angle K \geq m\angle L$

By angle – side relation $JL \geq JK$

This contradicts the given side lengths so the assumption must be false

Therefore, $m\angle K < m\angle L$