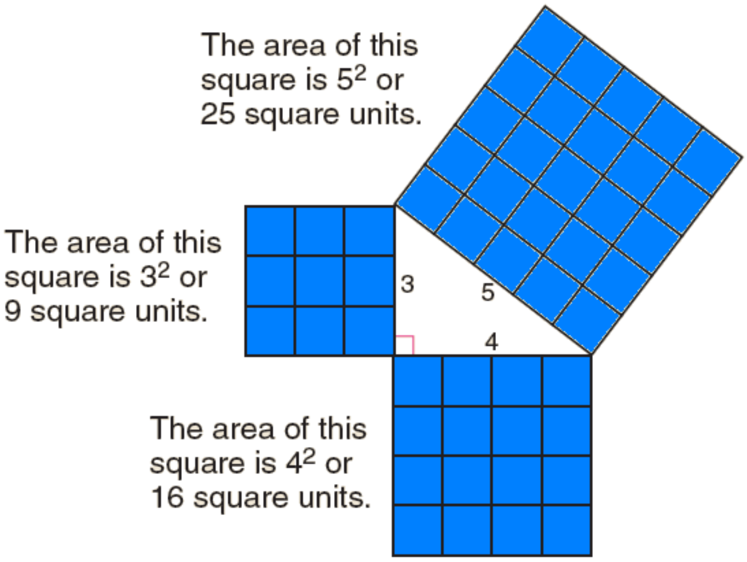
The Pythagorean Theorem

The relationship among 9, 16, and 25 forms the basis for the **Pythagorean Theorem**. It can be illustrated geometrically.



The sides of the right triangle have lengths of 3, 4, and 5 units. The area of the larger square is equal to the total area of the two smaller squares.

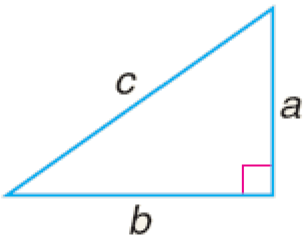
$5^2 = 3^2 + 4^2$

$25 = 9 + 16$

This relationship is true for *any* right triangle.

Theorem 1: **Pythagorean Theorem:** In a right triangle, the square of the length of the hypotenuse *c* is equal to the sum of the squares of the lengths of the legs *a* and *b*.

$$c^2 = a^2 + b^2$$



If two measures of the sides of a right triangle are known, the Pythagorean Theorem can be used to find the measure of the third side.

Example 1: Find the length of one leg of a right triangle if the length of the hypotenuse is 15 meters and the length of the other leg is 7 meters.
Hypotenuse is 15 meters $\Rightarrow$ c=15
Leg is 7 meters $\Rightarrow$ b=7
Using the Pythagorean Theorem: $c^2 = a^2 + b^2$

$\Rightarrow 15^2 = a^2 + 7^2$

$\Rightarrow 225 = a^2 + 49$

$\Rightarrow a^2 = 225 - 49$

$\Rightarrow a^2 = 225 - 49 = 176$

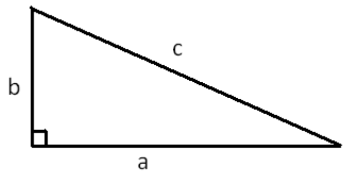
$\Rightarrow a = \sqrt{176} \approx 13.3$

The converse of the *Pythagorean Theorem* is also true.

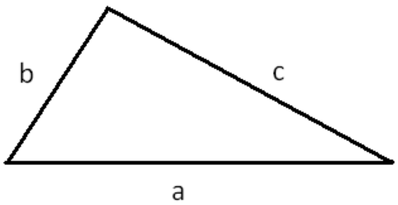
Theorem 2: **Converse of the Pythagorean Theorem**: If *c* is the measure of the longest side of a triangle, *a* and *b* are the lengths of the other two sides, and $c^2 = a^2 + b^2$, then the triangle is a right triangle.

Theorem 3: If a triangle has sides of lengths *a*, *b*, and *c* where *c* is the longest length and $c^2 = a^2 + b^2$, then the triangle is a right triangle with *c* its hypotenuse.

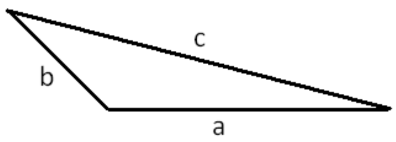
Theorem 4: If *a*, *b*, and *c* represent the lengths of the sides of a triangle, and *c* is the longest length, then the triangle is obtuse if $c^2 > a^2 + b^2$, and the triangle is acute if $c^2 < a^2 + b^2$.



$c^2 = a^2 + b^2$
$\Rightarrow$ Right Triangle



$c^2 < a^2 + b^2$
$\Rightarrow$ Acute Triangle



$c^2 > a^2 + b^2$
$\Rightarrow$ Obtuse Triangle