The Distance and Midpoint Formulas

<u>Midpoint Formula</u>

A midpoint is a point that denotes the middle of any given line segment.
In a Cartesian system the x – coordinate of the midpoint is the average of the x coordinates of the endpoints and the y – coordinate is the average of the y coordinates of the endpoints.

Rule 1: If the coordinates of A(x_1, y_1) and B(x_2, y_2), then the midpoint M of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

The midpoint formula is used when you need the point that is exactly in the middle between two other points. The midpoint formula is applied when you need to find a line that bisects a specific line segment.

Example 1: Find the coordinates (x, y) of the midpoint of the segment that connects the points $(-4, 6)$ and $(3, -8)$

$$x = \frac{x_1 + x_2}{2}$$

$$\Rightarrow \quad x = \frac{-4+3}{2} \Rightarrow \quad x = \frac{-1}{2}$$

$$y = \frac{y_1 + y_2}{2}$$

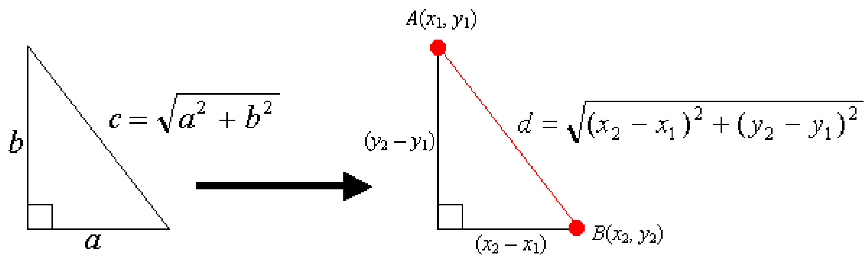
$$\Rightarrow \quad y = \frac{6 + (-8)}{2}$$

$$\Rightarrow \quad y = \frac{-2}{2} = -1$$

Therefore, the coordinates of the midpoint of the segment is $\left(-\frac{1}{2}, -1\right)$

<u>Distance Formula</u>

The distance formula can be obtained by creating a triangle and using the Pythagorean Theorem to find the length of the hypotenuse. The hypotenuse of the triangle will be the distance between the two points.



Rule 2: The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The advantage of the Distance Formula is that you do not need to draw a picture to find the answer you need the coordinates of the endpoints of the segment

Don't mismatch the *x*-values and *y*-values. Don't subtract an *x* from a *y*, make sure you've paired the numbers properly.

Don't get careless with the square-root symbol. The symbol should appear in each step.

The quantity inside the parenthesis should be simplified before you square it. So, all the answers should be positive because even the square of a negative number is positive.

Example 2: Find the distance between $(-2,3)$ and $(8,-1)$

Plug any given information into the distance equation.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow\ d = \sqrt{(8-(-2))^2 + (-1-3)^2}$$

$$\Rightarrow\ d = \sqrt{(10)^2 + (-4)^2}$$

$$\Rightarrow\ d = \sqrt{116}$$

Example 3: Verify that the points $(-3,4),(1,0) \, and \, (5,4)$ form a right triangle

$$a = \sqrt{(-3-1)^2 + (4-0)^2} = \sqrt{32}$$

$$b = \sqrt{(1-5)^2 + (0-4)^2} = \sqrt{32}$$

$$c = \sqrt{(-3-5)^2 + (4-4)^2} = \sqrt{64}$$

The triangle is a right triangle because:

$$a^2 + b^2 = \left(\sqrt{32}\right)^2 + \left(\sqrt{32}\right)^2 = 32 + 32 = 64 = c^2$$

Example 4: Consider the points $A(-4,5), B(-2,-1), C(5,0)$ *and* $D(3,6)$.
Show that ABCD is a parallelogram .

To prove ABCD is a parallelogram it's enough to prove that oppsite sides are equal.

So we want to prove $AB = DC$ and $BC = AD$,

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(-2+4)^2 + (-1-5)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}.$$

$$DC = \sqrt{(x_C - x_D)^2 + (y_C - y_D)^2} = \sqrt{(5-3)^2 + (0-6)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}.$$

$$BC = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{(5+2)^2 + (0+1)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}.$$

$$AD = \sqrt{(x_D - x_A)^2 + (y_D - y_A)^2} = \sqrt{(3+4)^2 + (6-5)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}.$$

So $AB = DC = 2\sqrt{10}$ and $BC = AD = 5\sqrt{2}$, ABCD is a parallelogram.