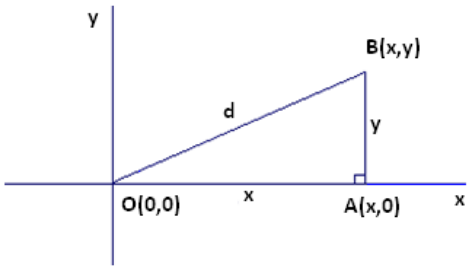
The Distance and Midpoint Formulas

How far apart are the two points (7,0) and (14,0)? Without even drawing a graph, you might be able to say they are 7 units apart. They're both on the x axis, so it's just a straight line measurement. But how can you measure the distance between two random points on a graph?

The distance formula is an application of Pythagoras' theorem for right triangles.

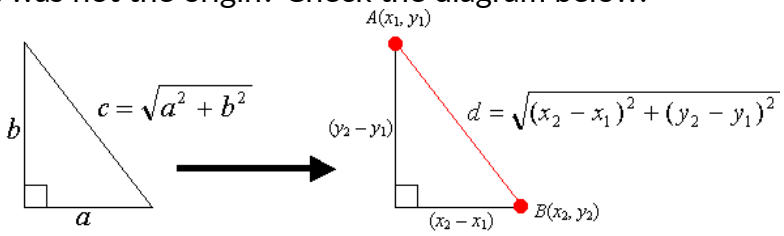
The distance of a point (x, y) from the origin



According to the Pythagorean Theorem $d^2 = x^2 + y^2$. Therefore, $d = \sqrt{x^2 + y^2}$

"The distance of a point from the origin is equal to the square root of the sum of the squares of the coordinates."

What if the first point was not the origin? Check the diagram below:



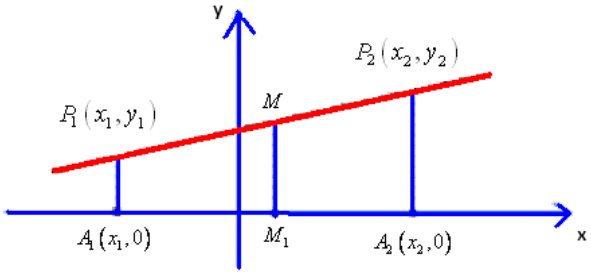
Rule 1: The distance formula: The distance *d* between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1: Find the distance between points *A*(1, 2) and *B*(- 3, 7).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-3-1)^2 + (7-2)^2} = \sqrt{(-4)^2 + (5)^2} = \sqrt{16+25} = \sqrt{41}$$

Midpoint of a segment

The line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ parallel to the y-axis intersect the x – axis at $A_1(x_1, 0)$ and $A_2(x_2, 0)$. The line through M parallel to the y – axis bisects the segment A_1A_2 at point M_1



M_1 is halfway from A_1 to A_2, the x – coordinate of M_1 is:

$$x_1 + \frac{1}{2}(x_2 - x_1) = x_1 + \frac{1}{2}x_2 - \frac{1}{2}x_1 = \frac{1}{2}x_1 + \frac{1}{2}x_2 = \frac{x_1 + x_2}{2}$$

Similarly for the y – value of the midpoint

Rule 2: **The Midpoint Formula:** If the coordinates of *A* and *B* are (x_1, y_1) and (x_2, y_2) respectively, then the midpoint, *M*, of *AB* is has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$