Tangent and Normal Lines

A **Tangent Line** is a line which locally touches a curve at one and only one point.
Talking about the line let us recall some of the terms together:

1) The slope-intercept formula for a line is $y = mx + b$, where m is the slope of the line and b is the y-intercept.
2) The point-slope formula for a line is $y - y_1 = m(x - x_1)$. This formula uses a point on the line, denoted by (x_1, y_1), and the slope of the line, denoted by m, to calculate the slope-intercept formula for the line.

Finding equations of a tangent line
One of the major applications of the derivative concept is to find the equation of the tangent and normal lines.

Consider the function: $y = f(x)$, with point (x_1, y_1) lying on the function graph.

Definition 1: The **tangent** line to the function at (x_1, y_1) is the straight line that touches $y = f(x)$ at that point. Both the graph of $y = f(x)$ and the tangent line pass through the point, and the tangent line has the same gradient, 'm', as the function at that point.

Definition 2: The **slope of the tangent** line is equal to the value of the derivative of the function at that point. Once we have the slope and one point then we can find the equation of the tangent line using the point – slope formula.

Definition 3: The **normal** line to function $y = f(x)$ at the point (x_1, y_1) is the straight line that passes through the point and perpendicular to the tangent line. The gradient of the normal line is $-\frac{1}{m}$, where 'm' is the gradient of the tangent line at the same point.

Here is a summary of the steps you use to find the equation of a tangent line to a curve at an indicated point:

1) Find the first derivative of f(x).
2) Plug x value of the indicated point into f '(x) to find the slope at x.
3) Plug x value into f(x) to find the y coordinate of the tangent point.
4) Combine the slope from step 2 and point from step 3 using the point-slope formula to find the equation for the tangent line.

Remember: The slope of the tangent equals the derivative of the function at the point of tangency. So stick the value given for x into the equation you have for y' to find that value. That is the "m" for the tangent line. Its negative reciprocal is the "m" for the normal line.

Example 1: Find the equation of the tangent line for the function $f(x)=x^2+1$ at point (3, 10).

Find the slope of the function by differentiation:

$f(x)=x^2+1 \Rightarrow f'(x)=2x$

Now, substitute x by 3 using the point (3, 10)

Plug both slope and point values into a linear equation

$(y-y_1)=m(x-x_1)$

$(y-10)=6(x-3)$

$y=6x-8$