Sum – to – Product Formulas

There are also formulas that combine a sum or difference into a product. The sum-to-product formulas are also used to prove the Law of Tangents, though that itself is no longer used in solving triangle.

Here's how to get the sum-to-product formulas. First make these definitions:

$A=\frac{1}{2}(u+v)$, and $B=\frac{1}{2}(u-v)$

Then you can see that

$A+B=u$, and $A-B=v$

Now make those substitutions in all four formulas of product to sum, and after simplifying you will have the sum-to-product formulas:

Rule 1: **Factorization (Sums or Differences into Products)**

1) $\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
2) $\sin\alpha-\sin\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$
3) $\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
4) $\cos\alpha-\cos\beta=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

OR

$\cos\alpha-\cos\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)$

Example 1: Verify that $\sin 70^0+\cos 40^0=\sqrt{3}\sin 80^0$

$\sin 70^0=\sin\left(90^0-20^0\right)=\cos 20^0$ [Since $\sin\left(90^0-\theta\right)=\cos\theta$]

$\Rightarrow \sin 70^0+\cos 40^0$

$=\cos 20^0+\cos 40^0$

$=2\cos\left(\frac{40^0+20^0}{2}\right)\cos\left(\frac{40^0-20^0}{2}\right)$

$=2\cos 30^0\cos 10^0$

$=2\bullet\left(\frac{\sqrt{3}}{2}\right)\cos 10^0$

$=\sqrt{3}\cos 10^0$

But $\sin\left(90^0-\theta\right)=\cos\theta \Rightarrow \cos 10^0=\sin\left(90^0-10^0\right)=\sin 80^0$

$\therefore \sqrt{3}\sin 80^0$