Name: _____

Sum – to – Product Formulas

Exercise 1: Use the sum to product formulas to write the sum or difference as a product

1) $\sin 5x - \sin 3x$

2) $\sin 3\theta + \sin \theta$

3) $\cos 6\theta + \cos 2\theta$

4) $\sin \theta + \sin 5\theta$

5) $\sin(\alpha + \beta) - \sin(\alpha - \beta)$

6) $\cos(\alpha + 2\pi) + \cos(\alpha)$

7) $\cos\left(\alpha + \frac{\pi}{2}\right) - \cos\left(\alpha - \frac{\pi}{2}\right)$

8) $\sin\left(\alpha + \frac{\pi}{2}\right) + \sin\left(\alpha - \frac{\pi}{2}\right)$

Exercise 2: Use the sum to product formulas to find the exact value of the expression.

1) $\sin 60^\circ + \sin 30^\circ$

2) $\cos 120^\circ + \cos 30^\circ$

3) $\cos\frac{3\pi}{4} - \cos\frac{\pi}{4}$

4) $\sin\frac{5\pi}{4} - \sin\frac{3\pi}{4}$

Exercise 3: Verify the identity

1) $\sin\left(\frac{\pi}{6} + x\right) + \sin\left(\frac{\pi}{6} - x\right) = \cos x$

2) $\cos\left(\frac{\pi}{3} + x\right) + \cos\left(\frac{\pi}{3} - x\right) = \cos x$

3) $\frac{\cos t + \cos 3t}{\sin 3t - \sin t} = \cos t$

4) $\frac{\cos 4x + \cos 2x}{\sin 4x + \sin 2x} = \cos 3x$

5) $\frac{\sin x \pm \sin y}{\cos x + \cos y} = \tan\frac{x \pm y}{2}$

6) $\frac{\sin x + \sin y}{\cos x - \cos y} = \cot\frac{x - y}{2}$

7) $\frac{\cos 4x + \cos 2x}{\sin 4x + \sin 2x} = \cot 3x$

8) $\frac{\cos t + \cos 3t}{\sin 3t - \sin t} = \cot t$

Exercise 4: Use sum-to-product identities to find the exact value each expression without using calculators.

1) $\cos 285^\circ + \cos 195^\circ$
2) $\cos 165^\circ - \cos 75^\circ$
3) $\cos 15^\circ - \cos 105^\circ$
4) $\sin 75^\circ - \sin 165^\circ$

Exercise 5: Simplify

1) $\dfrac{\sin 3\theta + \sin \theta}{\cos 3\theta + \cos \theta}$

2) $\dfrac{\sin 7\theta + \sin 5\theta}{\cos 7\theta + \cos 5\theta}$

3) $\dfrac{\sin 13A + \sin A}{\cos 13A + \cos A}$

Exercise 6: Find all solutions of the equation in the interval $(0, 2\pi)$. Use a graphing utility to graph the equation and verify the solutions.

1) $\sin 6x + \sin 2x = 0$
2) $\cos 2x - \cos 6x = 0$
3) $\dfrac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$
4) $\sin^2 3x - \sin^2 x = 0$