Square-Root Functions

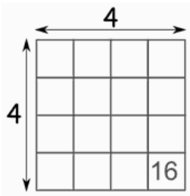
You learned that squaring a number means using that number as a factor twice. The opposite of squaring is finding a square root.
To find a square root of 49, you must find two equal factors whose product is 49.
$7 \times 7 = 49 \rightarrow$ The square root of 49 is 7

Definition 1: The square root of a number is a number which multiplied by itself, gives you the original number. Its symbol is $\sqrt{\ }$ called a radical sign.

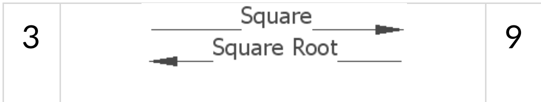
$$\sqrt{a} = b \Leftrightarrow b^2 = a$$

For example:

- $\sqrt{16} = 4$, because $4 \times 4 = 16$.



- 3 squared is 9, so the **square root of 9 is 3**



Fractional exponents can be used instead of using the radical sign ($\sqrt{\ }$). We use fractional exponents because often they are more convenient, and it can make algebraic operations easier to follow.

The nth root of a number can be written using the power 1/n, as follows: $a^{\frac{1}{n}} = \sqrt[n]{a}$
Meaning: The nth root of a when multiplied n times, gives us a.

$$a^{1/n} \times a^{1/n} \times a^{1/n} \times a^{1/n} \times a^{1/n} \times \ldots \times a^{1/n} = a$$

Definition 2: The number under the radical is called the radicand (in the above case, the number a), and the number indicating the root being taken is called the order (or index) of the radical (in our case n).

If we need to raise the n-th root of a number to the power m (say), we can write this as:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

In English, this means "take the nth root of the number, then raise the result to the power m".

Rule 1: For any positive value x, $\sqrt{x^2} = x$. Similarly, for any positive value x, $-\sqrt{x^2} = -x$

Remark: $\sqrt{(-x)^2} = \sqrt{(-x)(-x)} = \sqrt{x^2} = x$. Check the two examples below:

1) $\sqrt{(-5)^2} = \sqrt{(-5)(-5)} = \sqrt{5^2} = 5$ while $-\sqrt{5^2} = -5$
2) $\sqrt{(-14)^2} = \sqrt{(-14)(-14)} = \sqrt{14^2} = 14$ while $-\sqrt{14^2} = -14$

Square root of numbers that are not perfect squares would be irrational numbers. (Irrational numbers are numbers that cannot be written as fractions. In decimal form, these numbers go on forever and the same pattern of digits is not repeated.)

For example, $\sqrt{2}$ = 1.4142...

Usually, we would need to use a calculator to get the square root of a non-perfect square.

Rule 2: The quotient and product rule are important for simplifying rational expressions

1) **The product of two square roots of two positive numbers** *a* **and** *b* **is equal to the square root of product** *ab*, $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ for any positive numbers *a* and *b*.

2) **The quotient of the square roots of two positive numbers** *a* **and** *b*($b \neq 0$) **is equal to the square root of the quotient,** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}; b \neq 0$ for any positive numbers *a* and *b*($b \neq 0$).

Keep in mind that a square root of a given number multiplied by itself is the given number.

$\left(\sqrt{x}\right) \times \left(\sqrt{x}\right) = \left(\sqrt{x}\right)^2 = x$ for any positive number *x*.

Rule 3: The steps involved **in simplifying square roots using the perfect square method** are:

Step 1: Find the perfect square(s) that will divide the number in the square root.
Step 2: Write the number as a factor of the perfect square(s).
Step 3: Reduce the perfect squares.

Rule 4: Simplify square roots can be done also using the prime factorization method
The steps involved are:
Step 1: Break the number in the square root into prime factors
Step 2: For each pair of factors, “take one out” of the square root sign
Step 3: The remaining factors in the square root sign are multiplied together.