Solving Trigonometric Equations

A mathematical **equation** is an equality relationship involving mathematical quantities.
For example, the expression $x^2-x-1=5$ is an equation. Equations may contain one or more variables. The above equation, for example, contains the variable *x*. Many scientific and technological events require the **solution** of such equations. That is, it is necessary to determine the value(s) of a variable that make a given equation valid. This is called **solving** an equation. The above example is easily solved using algebraic techniques. First, consider

$x^2-x-1=5 \Rightarrow x^2-x-6=0 \Rightarrow (x+2)(x-3)=0$

Since the product of two numbers is zero only when one (or both) is zero, we see that
(*x* + 2) (*x* − 3) = 0 only if *x*+2 = 0 or *x* − 3 = 0. This means that the solutions are *x* = −2 *and* 3.

Definition1: A **trigonometric equation** is one that involves one or more trigonometric functions.
For example, $\sin^2 x+2=4$

The basic strategy for solving a trigonometric equation is to use trigonometric identities and algebraic techniques to reduce the given equation to an equivalent but more manageable expression.

Solutions for trigonometric equations follow no standard procedure, but there are a number of techniques that may help in finding a solution. These techniques are essentially the same as those used in solving algebraic equations, only now we are manipulating trigonometric functions: we can factor an expression to get different, more understandable expressions, we can multiply or divide through by a scalar, we can square or take the square root of both sides of an equation, etc. Also, using the basic identities, we can substitute certain functions for others, or break a function down into two different ones, like expressing tangent using sine and cosine.

Example 1: Solve $2\cos t=\sqrt{3}$

Divide both sides by 2 and then go to the unit circle.

$$2\cos t=\sqrt{3}$$

$$\Rightarrow \cos t=\frac{\sqrt{3}}{2}$$

Using your calculator $t=\cos^{-1}\frac{\sqrt{3}}{2}=\frac{\pi}{6}$

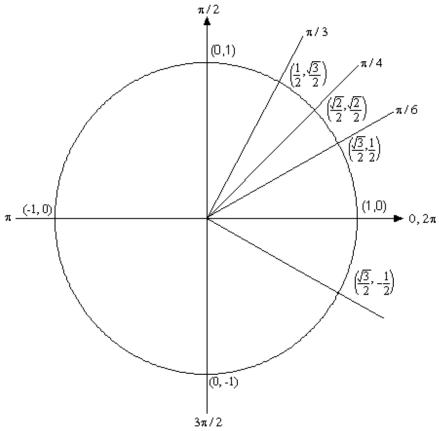
But $\cos\theta=\cos(2\pi-\theta)$

$$\Rightarrow t=2\pi-\frac{\pi}{6}=\frac{12\pi}{6}-\frac{\pi}{6}=\frac{11\pi}{6}$$

On the other hand the period of the cosine function is $2\pi n$
So, all together the complete solution to this problem is

$$t=\frac{\pi}{6}+2\pi n...............n=0,\pm1,\pm2,\pm3,\pm4,.....$$

$$t=\frac{11\pi}{6}+2\pi n...............n=0,\pm1,\pm2,\pm3,\pm4,.....$$



Let us study some of the famous cases.

Case 1: **Trigonometric Equations involving Powers:**
When the trig function has a power, it will have to be solved by extracting square roots or factoring.

Case 2: **Solving Quadratic Equations:**
Remember to first solve for the trig function and then solve for the angle value.

Case 3: **Using Identities in Equation Solving:**
If there is more than one trig function in the equation,
identities are needed to reduce the equation to a single function for solving.