Solving Linear Systems by Linear Combinations

Use elimination when the coefficients of one of the variables are equal or additive inverses. For example, consider the system below.

$5x + 11y = 12$ $\qquad$ $2x + 11y = 36$

Notice that the coefficient of y in both equations is the same. You can solve this system of equations in three steps.

Step 1: Subtract the two equations, so that y is eliminated. The result is an equation only in x: $3x = -24$.
Step 2: The next step is to solve for x: $x = -8$.
Step 3: Substitute the value of x in any one of the two original equations and solve for y: $y = \frac{52}{11}$

Rule 1: The solution of a system can be found by using an algebraic method called the Elimination Method. To find the set of solutions using the substitution method, we follow the steps listed below:

Step 1: Simplify and put both equations in the form $Ax + By = C$ if needed.
Step 2: Multiply one or both equations by a number that will create opposite coefficients for either x or y if needed.
Step 3: Add equations.
Step 4: Solve for remaining variable.
Step 5: Solve for second variable.
Step 6: Check the proposed ordered pair solution in BOTH original equations.

Example 1: Use elimination method to solve the system of equations

$$\begin{cases} 2x+3y=7 \\ x+3y=8 \end{cases}$$

$$\begin{array}{rl} & 2x+3y=7 \\ (-) & x+3y=8 \\ \hline & x+0=-1 \\ & x=-1 \end{array}$$

Now substitute in either equation $x=-1$ to find the value of y.

$x+3y=8$
$\Rightarrow -1+3y=8$
$\Rightarrow 3y=9$
$\Rightarrow y=3$

The solution of the system of equations is $(-1,3)$

Check:

$2x+3y=7$ substitute $(-1,3)$
$2x+3y=2(-1)+3(3)=-2+9=7$ True

$x+3y=8$ substitute $(-1,3)$
$x+3y=(-1)+3(3)=-1+9=8$ True