# Solving Linear Inequalities

You already know that equations are mathematical statements that describe two expressions with equal values. When the values of the two expressions are *not* equal, their relationship can be described in an inequality.
Verbal phrases like *greater than* or *less than* describe inequalities. For example, 6 is *greater than* 2. This is the same as saying 2 is *less than* 6.
The chart below lists other phrases that indicate inequalities and their corresponding symbols.

| Inequalities | | | |
|---|---|---|---|
| < | ≥ | > | ≤ |
| • Less than<br>• Fewer than | • Less than or equal to<br>• At most<br>• No more than<br>• A maximum of | • Greater than<br>• More than | • Greater than or equal to<br>• At least<br>• No less than<br>• A minimum of |

Example 1: Suppose the minimum driving age in your state is 16. Write an inequality to describe people who are *not* of legal driving age in your state.

Let *d* represents the ages of people who are *not* of legal driving age.

$\underbrace{\text{The ages of all drivers}}_{d}\ \underbrace{\text{are greater than or equal to}}_{\geq}\ \underbrace{\text{16 years}}_{16}.$

$d \geq 16$ is the same as $16 \leq d$

Let $d'$ represents the ages of people who are *not* of legal driving age.

Then $d'$ is less than 16, or $d' < 16$

Not only can inequalities be expressed through words and symbols, but they can also be graphed.

Rule 1: Addition and Subtraction Properties for Inequalities: For any inequality, if the same quantity is added or subtracted to each side, the resulting inequality is true.
Symbols: For all numbers *a*, *b*, and *c*,
1. if $a > b$, then $a + c > b + c$ and $a - c > b - c$.
2. if $a < b$, then $a + c < b + c$ and $a - c < b - c$.

Rule 2: Multiplication Property for Inequalities: If you multiply each side of an inequality by a positive number, the inequality remains true. If you multiply each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.

**Symbols:** For all numbers *a*, *b*, and *c*,
**1.** if *c* is positive and $a > b$, then $ac > bc$, and if *c* is positive and $a < b$, then $ac < bc$.
**2.** if *c* is negative and $a > b$, then $ac < bc$, and if *c* is negative and $a < b$, then $ac > bc$.

Rule 3: Division Property for Inequalities: If you divide each side of an inequality by a positive number, the inequality remains true. If you divide each side of an inequality by a negative number, the **inequality symbol must be reversed for the inequality to remain true.**

**Symbols:** For all numbers *a*, *b*, and *c*,

**1.** If *c* is positive and *a* < *b*, then $\frac{a}{c} < \frac{b}{c}$, and if *c* is positive and *a* > *b*, then $\frac{a}{c} > \frac{b}{c}$

**2.** If *c* is negative and *a* < *b*, then $\frac{a}{c} > \frac{b}{c}$ and if *c* is negative and *a* > *b*, then $\frac{a}{c} < \frac{b}{c}$

Some inequalities involve more than one operation. The best strategy to solve multi-step inequalities is to undo the operations in reverse order. In other words, work backward just as you did to solve multi-step equations.

For example, $3x - 9 \geq 12$ is a multi-step inequality; you can solve this inequality by following these steps.

To solve $3x - 9 \geq 12$, we **undo addition** first:

$$3x - 9 \geq 12$$
$$\Rightarrow 3x - 9 + 9 \geq 12 + 9$$
$$\Rightarrow 3x \geq 21$$

Now, we **undo multiplication**:

$$3x \geq 21$$
$$\Rightarrow \frac{3x}{3} \geq \frac{21}{3}$$
$$\Rightarrow x \geq 7$$

Definition 1: **Compound inequalities** are two inequalities considered together.

A. A compound inequality containing the word and is true if and only if *both* inequalities are true. This type of compound inequality is called a **conjunction**. The graph of a compound inequality using *and* is the intersection of the graphs of the two inequalities.

   Examples of conjunctions:
- x > -5 and x <1
- y < 3 and y > -3

B. A compound inequality containing the word or is true if either of the inequalities is true (if one or more of the inequalities is true). This type of compound inequality is called a **disjunction**. The graph of a compound inequality using *or* is the union of the graphs of the two inequalities.

   Examples of disjunctions:
- x > -5 or x > 1
- y < 3 or y > -3