

Solving Equations by Factoring

Examine the equation below: $ab = 0$

If you let $a = 3$, then logically b must equal 0. Similarly, if you let $b = 10$, then a must equal 0.

Now try letting a be some other non-zero number. You should observe that as long as a does not equal 0, b must be equal to zero.

To state the observation more generally, "If $ab = 0$, then either $a = 0$ or $b = 0$." This is an important property of zero which we exploit when solving by factoring.

0 is our magic number because the only way a product can become 0 is if at least one of its factors is 0.

When the example is factored into $(x - 2)(x - 3) = 0$, this property was applied to determine that either $(x - 2)$ must equal zero, or $(x - 3)$ must equal zero. Therefore, we are able to create two equations and determine two solutions from this observation.

Remark:

You can't guarantee what the factors would have to be if the product was set equal to any other number. For example if $ab = 1$, then $a = 5$ and $b = 1/5$ or $a = 3$ and $b = 1/3$, etc. But with the product set equal to 0, we can guarantee finding the solution by setting each factor equal to 0.

Example 1 : Solve: $4x(2x - 1) = 0$

To solve this equation, find values of x that make the product $4x(2x - 1)$ equal to 0. Since the product of 0 and any number is 0, *at least one* of the factors in the expression must be zero.

$$4x = 0 \text{ or } 2x - 1 = 0$$

$$x = 0 \text{ and } x = \frac{1}{2}$$

The solutions of $4x(2x - 1) = 0$ are 0 and $\frac{1}{2}$.

This method of solving equations uses the Zero Product Property.

Rule 1: Zero Product property: For all numbers a and b , if $ab = 0$, then $a = 0$, $b = 0$ or both a and b equal 0.

We can use this property to solve any equation that is written in the form $ab = 0$.

Rule 1: Solving by Factoring process will require five major steps:

Step1: Move all terms to one side of the equation, usually the left, using addition or subtraction.

Step2: Factor the equation completely.

Step3: Set each factor equal to zero, and solve.

Step4: List each solution from Step 3 as a solution to the original equation.

Step5: Check the solutions in the original equation