Similar Triangles

The pair of towers pierce the sky over Dubai with spires over 1,000 feet tall. The taller tower's main function is to hold offices. The lesser tower is mostly taken up with a hotel.

The strength of the towers is conveyed through the concrete columns on the three corners framing the remainder of the facade which is primarily glass. As you notice from the pictures that the top part consist of triangles that are similar because they have the same shape but different sizes.

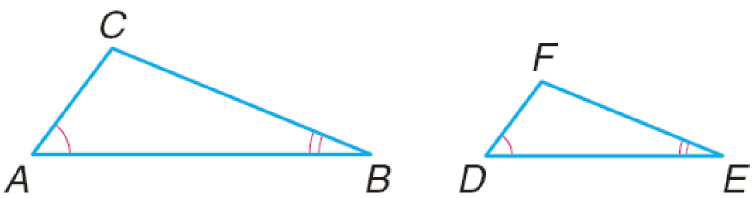


We learned several basic tests for determining whether two triangles are congruent. Recall that each congruence test involves only three corresponding parts of each triangle. Likewise, there are tests for similarity that will not involve all the parts of each triangle.

Postulate 1: Angle-Angle (AA) Similarity

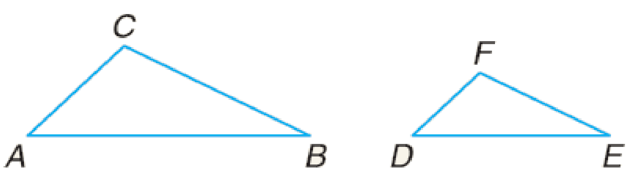
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

$$\begin{aligned}&\text{If } \angle A \cong \angle D\\&\text{and } \angle B \cong \angle E\\&\text{, then } \triangle ABC \sim \triangle DEF\end{aligned}$$



Theorem 1: Side-Side-Side (SSS) Similarity
If all the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

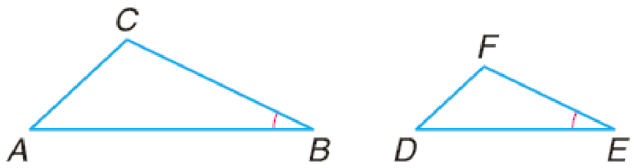
$$\text{If } \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$
$$\text{, then } \triangle ABC \sim \triangle DEF$$



Theorem 2: Side-Angle-Side (SAS) Similarity

If the measures of two sides of a triangle are proportional to the measures of two corresponding side of another triangle and the included angles are congruent,
then the triangles are similar.

$$\text{If } \frac{AB}{DE} = \frac{BC}{EF}$$
$$\text{and } \angle B \cong \angle E$$
$$\text{, then } \triangle ABC \sim \triangle DEF$$

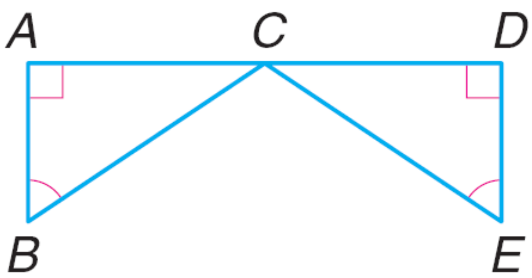


Example 1:

Given: $\angle B \cong \angle E$

$\angle A$ and $\angle D$ are right angles

Prove: $\frac{BC}{EC} = \frac{AB}{DE}$



Proof:

Statements	Reasons
1) $\angle B \cong \angle E$	1) Given
2) $\angle A$ and $\angle D$ are right angles	2) Given
3) $\angle A \cong \angle D$	3) All right angles are congruent
4) $\triangle ABC \sim \triangle DEC$	4) AA Postulate
5) $\frac{BC}{EC} = \frac{AB}{DE}$	5) 2 $\triangle$'s are similar $\Rightarrow$ sides are proportional

Theorem 3: Similarity of triangles is reflexive, symmetric, and transitive

Reflexive: $\Delta ABC \sim \Delta ABC$

Symmetric: If $\Delta ABC \sim \Delta DEF$, then $\Delta DEF \sim \Delta ABC$

Transitive: If $\Delta ABC \sim \Delta DEF$ and $\Delta DEF \sim \Delta GHI$, then $\Delta ABC \sim \Delta GHI$