Name: ____

Rolle's Theorem and Mean Value Theorem

Exercise 1: Use Rolle's theorem to show that the function $f(x) = -x^3 + 2x + 2$ has exactly one zero in the given interval $[1, 2]$

Exercise 2: Find the value(s) of *c* that satisfy Rolle's Theorem for $f(x) = -x^2 + 6x - 4$ on the interval [1, 5].

Exercise 3: Find the value(s) of *c* that satisfy the Mean Value Theorem for $f(x) = x^2 - 8x + 15$ on the interval [4, 6].

Exercise 4: Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

1. $f(x) = x^2 - 4x + 1, \quad [0,4]$
2. $f(x) = x^3 - 3x^2 + 2x + 5, \quad [0,2]$
3. $f(x) = x\sqrt{x+6}, \quad [-6,0]$

Exercise 5: Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but there is no number c in $(-1,1)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

Exercise 6: Verify Rolle's Theorem

1) $f(x) = x^4 - 4x^2 on[-2, 2]$
2) $f(x) = x^3 - 9x \ on[-3, 3]$

Exercise 7: Verify the Mean Value Theorem

1) $f(x) = x^4 on[0, 2]$
2) $f(x) = x^3 - 3x^2 on[-1, 3]$