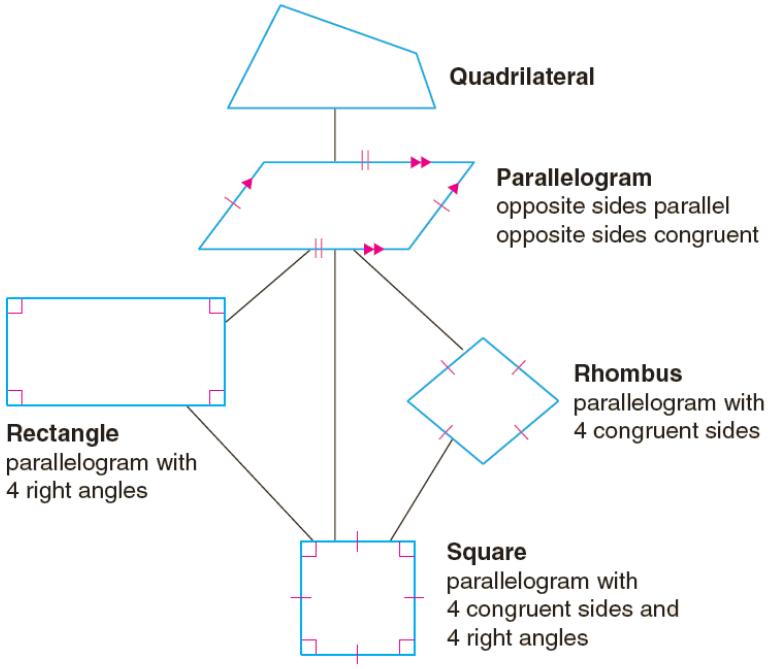
Rectangles, Rhombi, and Squares

We studied the properties of parallelograms. Now you will learn the properties of three other special types of parallelograms: **rectangles**, **rhombi**, and **squares**. The diagram shows how these quadrilaterals are related.

Notice how the diagram goes from the most general quadrilateral to the most specific one. Any four-sided figure is a quadrilateral. But a parallelogram is a special quadrilateral whose opposite sides are parallel.

The opposite sides of a square are parallel, so a square is a parallelogram.

In addition, the four angles of a square are right angles, and all four sides are equal. A rectangle is also a parallelogram with four right angles, but its four sides are not equal.



Both squares and rectangles are special types of parallelograms. The best description of a quadrilateral is the one that is the most specific.

Rectangles, rhombi, and squares have all of the properties of parallelograms. In addition, they have their own properties.

Part A: Rectangles

Definition 1: A **rectangle** is a parallelogram with four right angles.

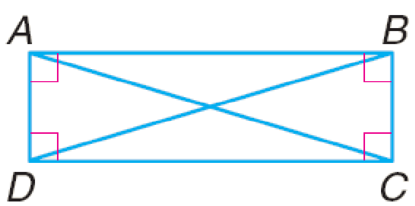
A rectangle has all of the properties of the parallelogram. In addition to that a rectangle has 4 right angles and congruent diagonals.

Theorem 1: A quadrilateral is a rectangle if and only if it has four right angles

Theorem 2: A parallelogram is a rectangle if and only if its diagonals are congruent.

ABCD is a rectangle

$\Leftrightarrow \overline{AC} \cong \overline{BD}$



Theorem 3: If a parallelogram has one right angle then it is a rectangle.

Part B: Rhombi

Definition 2: A **rhombus** is a parallelogram with four congruent sides.

A rhombus has all of the properties of the parallelogram. In addition to that a rhombus has 4 congruent sides; the diagonals bisect the angles and are perpendicular

Theorem 4: A quadrilateral is a rhombus if and only if it has four congruent sides.

Theorem 5: If a parallelogram has two consecutive sides congruent, it is a rhombus.

Theorem 6: A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

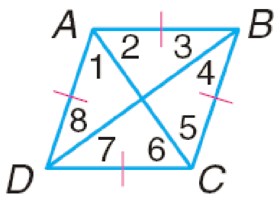
ABCD is a rhombus

$\Rightarrow m\angle 1 = m\angle 2$

$\Rightarrow m\angle 3 = m\angle 4$

$\Rightarrow m\angle 5 = m\angle 6$

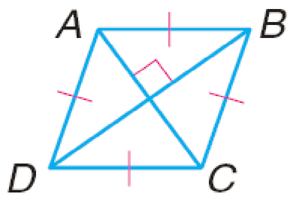
$\Rightarrow m\angle 7 = m\angle 8$



Theorem 7: A parallelogram is a rhombus if and only if the diagonals are perpendicular.

ABCD is a rhombus

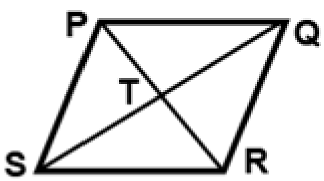
$\Leftrightarrow \overline{AC} \perp \overline{BD}$



Example 1:

Given: PQRS is a rhombus

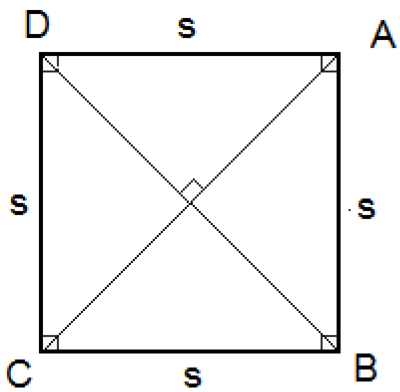
Prove: $\overline{PR} \perp \overline{SQ}$



Statements	Reasons
1) $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{PS}$	1) Def. of a rhombus
2) $\overline{QS}$ bisects $\overline{PR}$ at T	2) $\overline{QS}$ and $\overline{PR}$ are diagonals
3) $\overline{PT} \cong \overline{RT}$	3) In a rhombus, diagonals bisect each other
4) $\overline{QT} \cong \overline{QT}$	4) Reflexive property
5) $\triangle PQT \cong \triangle RQT$	5) SSS Postulate
6) $\angle QTP \cong \angle QTR$	6) CPCTC
7) $\angle QTP$ is a right angle	7) 2 $\cong$ $\angle$'s that form a linear pair are right $\angle$'s
8) $\overline{PR} \perp \overline{SQ}$	8) Def. of perpendicular lines

Part C: Squares

Definition 3: A square is a parallelogram with four congruent sides and four right angles.



A square is defined as a parallelogram with four congruent angles and four congruent sides. This means that a square is not only a parallelogram, but also a rectangle and a rhombus.

Therefore, A square has all of the properties of the parallelogram AND the rectangle AND the rhombus.

Theorem 8: A quadrilateral is a square if and only if it is a rhombus and a rectangle at the same time.

Questions you should be able to answer

1) Is every square a rectangle?
2) Is every rectangle a square?
3) Is every rhombus a square?
4) Is every square a rhombus?
5) Is every rectangle a rhombus?
6) Is every rhombus a rectangle?
7) Is every rhombus/square/rectangle a parallelogram?
8) Is every parallelogram a square or rectangle or rhombus?