Rational Expressions

The quotient of two polynomials is called a rational expression. Rational expressions are to polynomials what rational numbers are to integers.
The goal here is to learn to do everything that you do with Rational Numbers with rational expressions. That isn't too difficult as long as you know two things well, rational number arithmetic and polynomial factoring.

Example 1: Some examples of rational expressions

$$\frac{x^2-1}{2-x} \qquad \frac{1}{x} \qquad \frac{x^3+2x^2-x+7}{x^2-11x+3}$$

Excluded Values

Whenever an expression containing variables is present in the denominator of a fraction, you should be alert to the possibility that certain values of the variables might make the denominator equal to zero, which is forbidden. This means that when we are talking about rational expressions we can no longer say that the variable represents "any real number." Certain values may have to be excluded. For example, in the expression $\frac{3x+5}{7x}$, we cannot allow the value $x = 0$ so x must be $\neq 0$ for the rational expression to be defined, and for $\frac{3x+5}{x-3}$ we would say ($x \neq 3$) .

In the case of $\frac{4x+11}{x^2-9}$ we would exclude both $x = 3$ and $x = -3$, since either choice would make the denominator zero. Therefore, $\frac{4x+11}{x^2-9}$ is defined when $x \neq \pm 3$

Remark 1:

The restriction is on the denominator only.there is no restriction on the numerator, but why? If the numerator is zero, that just makes the whole rational expression zero.
(assuming, of course, that the denominator is not zero), just as with common fractions. Recall that $\frac{0}{9}=0$, but $\frac{9}{0}$ is undefined.

<u>Simplifying rational expressions</u>

Simplification is also called reducing to lowest terms with number fractions. When you reduce fractions to lowest terms you divide top and bottom by common factors. With algebraic fractions you do the same thing, and you must remember that it is only common **factors** that you can divide out.

To simplify a rational expression:
Step 1: Factorize the numerator and denominator completely.
Step 2: Mention all the excluded values of x (the values that make the denominator equal to zero)
Step 3: Cross out any factors that are the same up and down.

When we reduce a fraction such as $\frac{6}{12} \to \frac{1}{2}$ we do so by noticing that there is a factor common to both the numerator and the denominator (a factor of 6 in this example), which we can divide out of both the numerator and the denominator: $\frac{6}{12} = \frac{1\times 6}{2\times 6} = \frac{1}{2}$

We use exactly the same procedure to reduce rational expressions:

$$\frac{4xy^2}{8x^2y} = \frac{4xy \bullet y}{4xy \bullet 2x} = \frac{\cancel{4xy} \bullet y}{\cancel{4xy} \bullet 2x} = \frac{y}{2x}$$

Keep in mind that <u>each term</u> in the numerator must have a factor that cancels a common factor in the denominator: $\frac{8x+6y}{2y} = \frac{\cancel{2}(4x+3y)}{\cancel{2}y} = \frac{4x+3y}{y}$

But simplifying $\frac{3x+1}{3x}$ ***like this*** $\frac{3x+1}{3x} = \frac{\cancel{3x}+1}{\cancel{3x}} = 1$ ***is completely wrong, can you tell why?***

WARNING: You can only cancel a factor of the <u>entire</u> numerator with a factor of the <u>entire</u> denominator

<u>Remark 2:</u>
A fraction with more than one term in the numerator can be split up into separate fractions with each term over the same denominator; then each separate fraction can be reduced if possible:

$$\frac{a \pm b}{d} = \frac{a}{d} \pm \frac{b}{d}$$

That is, $\frac{3x+1}{3x} = \frac{3x}{3x} + \frac{1}{3x} = 1 + \frac{1}{3x}$