Rates of Change

Displacement, Velocity, and Acceleration
Given an equation for the displacement of a moving object, you are can find an equation for its velocity and an equation for its acceleration, and use the equations to analyze the motion.
If x is the displacement of a moving object from a fixed plane (such as the ground), and t is time, then

Velocity: $v = x' = \frac{dx}{dt}$

Acceleration: $a = v' = \frac{dv}{dt} = x'' = \frac{d^2x}{dt^2}$

Speed: $|v|$

Example 1: Suppose a football is punted into the air. As it rises and falls, its displacement (directed distance) from the ground is a function of the number of seconds since it was punted.
$y = -16t^2 + 37t + 37$
where y is the football's displacement in feet and t is the number of seconds since it was punted. Find the velocity and the acceleration.

The velocity of the ball gives its speed. Because velocity is the instantaneous rate of change, it is a derivative.
$velocity = \frac{dy}{dt} = y' = -32t + 37$
The acceleration is constant, -32 ft/sec^2, for an object acted on only by gravity.

To tell quickly whether an object is speeding up or slowing down, compare the signs of the velocity and acceleration.

Definition: **Speeding Up or Slowing Down**
If velocity and acceleration have the same sign, the object is speeding up.
If velocity and acceleration have different signs, the object is slowing down.

The method of finding the rate of change consists of five steps:
Step1: Draw a figure and label the quantities that vary.
Step 2: Identify the rates of change that are known and the rate of change that is to be found.
Step 3: Find an equation that relates the quantity whose rate of change is to be found to the quantities whose rates of change are known.
Step 4: Differentiate both sides of this equation with respect to time and solve the derivative that will give the unknown rate of change.
Step 5: Evaluate this derivative at the appropriate point

A Physical Interpretation of the Mean Value Theorem

Recall that if $s = s(t)$ is the position, then the quotient $\frac{s(b)-s(a)}{b-a}$ is the average velocity of the object on the time interval $[a, b]$. Assuming $s = s(t)$ is continuous on $[a, b]$ and differentiable on (a, b), then by the Mean Value Theorem there is a number c in (a, b) such that $\frac{s(b)-s(a)}{b-a} = s'(c) = v(c)$

In other words, there is a time *c* when the instantaneous velocity is equal to the average velocity.

Example 2: The position of a particle on a line is given by $s(t) = t^3 - 3t^2 - 6t + 5$, where t is measured in seconds and s is measured in feet. Find

(a) The velocity of the particle at the end of 2 seconds.

(b) The acceleration of the particle at the end of 2 seconds.

(a) The velocity of the particle is

$v = s'(t) = 3t^2 - 6t - 6$

At $t = 2$ seconds $s'(2) = 3(2)^2 - 6(2) - 6$

$s'(2) = -6$ ft/sec

(b) The acceleration of the particle is

$a = v'(t) = s''(t) = 6t - 6$

At $t = 2$ seconds $v'(2) = s''(2) = 6(2) - 6$

$v'(2) = s''(2) = 6$ ft/ sec^2