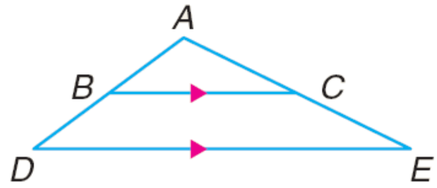
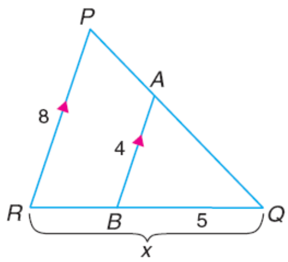
Proportions and Similar Triangles

Theorem 1: If a line is parallel to one side of a triangle and intersects the other two sides, then the triangle formed is similar to the original triangle.

If $\overline{BC} \parallel \overline{DE}$, then $\triangle ABC \sim \triangle ADE$



Example 1: In the figure, $\overline{AB} \parallel \overline{PR}$. Find the value of *x*.

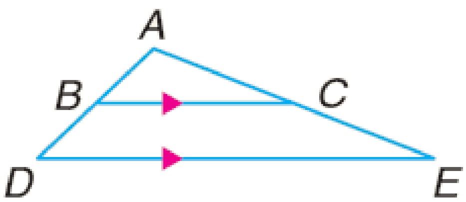


Statements	Reasons
1) $\overline{AB} \parallel \overline{PR}$	1) Given
2) $\triangle FJK \sim \triangle FGH$	2) Theorem 1
3) $\frac{AB}{PR} = \frac{QB}{QR}$	3) 2 $\triangle$'s are similar $\Rightarrow$ sides are proportional
4) $(AB)(QR) = (PR)(QB)$	
$(4)(x) = (4)(5)$	
$x = 5$	

Theorem 2: Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it separates the sides into segments of proportional lengths.

If $\overline{BC} \parallel \overline{DE}$, then $\frac{AB}{BD} = \frac{AC}{CE}$

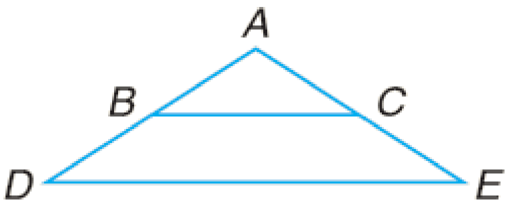


The converse of this theorem is also true.

Theorem 3: Converse of the Triangle Proportionality Theorem:

If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

If $\frac{AB}{BD} = \frac{AC}{CE}$, then $\overline{BC} \parallel \overline{DE}$



In particular if $\frac{AB}{BD} = \frac{AC}{CE} = \frac{1}{2}$ the then the points B and C are the midpoints of $\overline{AD}$ and $\overline{AE}$ respectively.
And $\overline{BC}$ is the midsegment of $\triangle ADE$

Special Segments of Similar Triangles:

Theorem 4: If two triangles are similar, then the measures of the *corresponding* ***altitudes*** are proportional to the measures of the corresponding sides

Theorem 5: If two triangles are similar, then the measures of the *corresponding* ***angle bisectors*** of the triangles are proportional to the measures of the corresponding sides

Theorem 6: If two triangles are similar, then the measures of the *corresponding* ***medians*** are proportional to the measures of the corresponding sides

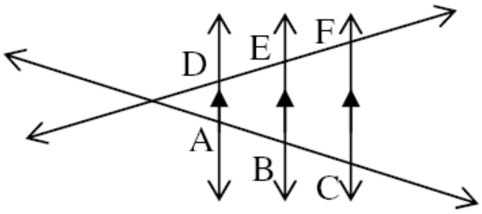
Theorem 7: **Proportional Perimeters Theorem:** If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides

Dividing Segments Proportionally

Corollary 1: PROPORTIONAL LENGTHS THEOREM: If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally

If $\overleftrightarrow{AD} \parallel \overleftrightarrow{EB} \parallel \overleftrightarrow{FC}$,

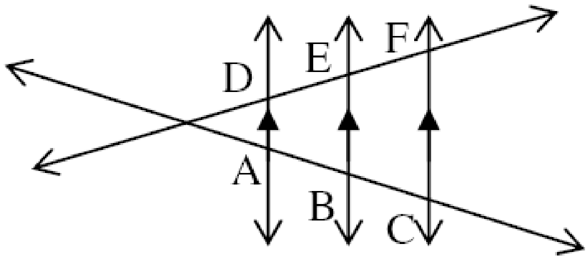
$$then \frac{AB}{BC} = \frac{DE}{EF}$$



Corollary 2: If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

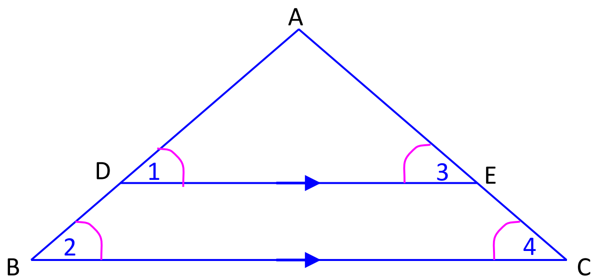
If $\overleftrightarrow{AD} \parallel \overleftrightarrow{EB} \parallel \overleftrightarrow{FC}$ and $\overline{BC} \cong \overline{AB}$

$then\, \overline{DE} \cong \overline{EF}$



Theorem 8: *THALES' THEOREM* (Side Splitting Theorem)

A line parallel to one side of a triangle divides the other two sides proportionally.



Given: Δ ABC; DE // BC

Prove: $\frac{BD}{AD} = \frac{CE}{AE}$

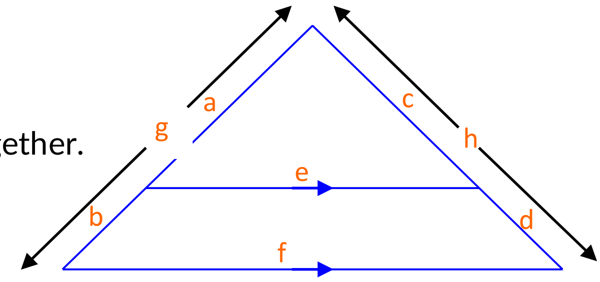
Proof:

STATEMENTS	REASONS
1) Δ ABC; DE // BC	1) Given
2) $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$	2) corresponding $\angle$s
3) Δ ABC ~ Δ ADE	3) AA similarity theorem
4) $\frac{AB}{AD} = \frac{AC}{AE}$	4) Def. of ~ Δ's.
5) AB = AD + BD; AC = AE + CE	5) Segment Addition Postulate
6) $\frac{AD + BD}{AD} = \frac{AE + CE}{AE}$	6) Substitution
7) $1+\frac{BD}{AD} = 1+\frac{CE}{AE}$	7) Division
8) $\frac{BD}{AD} = \frac{CE}{AE}$	8) Subtraction Property

Theorem 9: THALES' THEOREM AND SIMILAR TRIANGLES:

A straight line drawn parallel to one side of a triangle results in many proportions.

Be careful not to mix these proportions together.



Proportions resulting from similar triangles: $\frac{a}{g} = \frac{c}{h} = \frac{e}{f}$

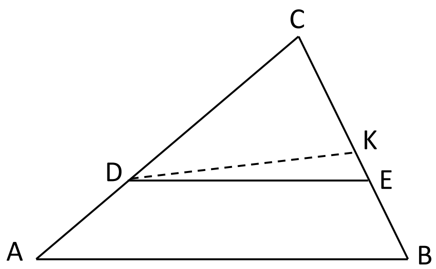
Proportions resulting from Thales' Theorem: $\frac{a}{b} = \frac{c}{d}$; $\frac{b}{g} = \frac{d}{h}$

Theorem 10: CONVERSE OF THALES' THEOREM: If a line divides two sides of a triangle proportionally, it is parallel to the third side.

Given: ΔABC with DE so drawn that $\frac{AC}{DC} = \frac{BC}{EC}$

Prove: $DE \,//\, AB$

Proof:



STATEMENTS	REASONS
Draw *DK // AB*	
1) ΔABC with DE so drawn that $\frac{AC}{DC} = \frac{BC}{EC}$	1) Given
2) $\frac{AC}{DC} = \frac{BC}{KC}$	2) Side Splitting Theorem
3) $\therefore EC = KC$	3) If three quantities of one proportion are equal respectively to 3 quantities of another, the 4th quantities are equal
4) $\therefore K$ coincides with *E*	4) Both points are equidistant from C on CB
5) $\therefore DK$ coincides with *DE*	5) only one line can be drawn through 2 pts.
6) $DE \,//\, AB$	6) A line assumes all the properties of the line with which it coincides