

Properties of Sets

Properties of Sets: The set operations verify the following properties:

Law 1: Associative Laws:

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Law 2: Commutative Laws:

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

Example 1: Prove $A \cap B = B \cap A$

$$A \cap B = \{x : x \in A \text{ and } x \in B\} = \{x : x \in B \text{ and } x \in A\} = B \cap A$$

Law 3: Distributive Laws:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Law 4: Identity Laws:

$$A \cup \emptyset = A$$

$$A \cap U = A$$

Law 5: Complement Laws:

$$A \cup \bar{A} = U$$

$$A \cap \bar{A} = \emptyset$$

Law 6: Idempotent Laws:

$$A \cap A = A$$

$$A \cup A = A$$

Law 7: Bound Laws:

$$A \cup U = U$$

$$A \cap \emptyset = \emptyset$$

Law 8: Absorption Laws:

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

Law 9: Involution Law:

$$\overline{\overline{A}} = A$$

Law 10: 0/1 laws:

$$\overline{\emptyset} = U$$

$$\overline{U} = \emptyset$$

Law 11: DeMorgan's Laws:

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

Even though the proofs of set equations are simple, they can be quite tedious, and we shall introduce a new tool for tackling problems of this sort.

If A and B are sets, then for an arbitrary element x of our universe of discourse U there are four possibilities:

$x \in A$ and $x \in B$, $x \in A$ and $x \notin B$, $x \notin A$ and $x \in B$, $x \notin A$ and $x \notin B$.

For every one of these cases let us consider if x is in the intersection of A and B : If $x \in A$ and $x \in B$, then $x \in A \cap B$. In all three other cases, we have $x \notin A \cap B$.

This observation can be put in the form of a table which looks like this:

A	B	$A \cap B$	$A \cup B$
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

Example 2: Using the table method, prove:

The second law of absorption $A \cap (A \cup B) = A$

A	B	$A \cup B$	$A \cap (A \cup B)$
1	1	1	1
1	0	1	1
0	1	1	0
0	0	0	0

Example 3: Using the table method, prove:

The first distributive law $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is:

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0