Probability

Let a random experiment have sample space S. Any assignment of probabilities to events must satisfy some basic laws of probability:

1) For any event A, P(A) ≥ 0.
2) P(S) = 1. Where S is the sample space
3) $0 \le P(A) \le 1$

Definition 1: The **theoretical probability** of an event is defined as the ratio:

$$\frac{\text{number of favorable cases}}{\text{total number of possible cases}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

Definition 2: **S = Sample space** is the set of all possible outcomes of a statistical experiment.

Example 1: Rolling a die. Rolling a 2 is a simple event (a possible outcome). The sample space would be rolling a 1, 2, 3, 4, 5, or 6.

Definition 3: **An event** is any subset of the sample space.

Notes:

- Since $\phi \subset S$ and S $\subset$ S, both ϕ = { } and S are events.
- ϕ = { } is called the impossible event and has probability zero, i.e., P(ϕ) = 0.
- S is called the definite event and has probability of 1, i.e., P(S) = 1.
- Probability of any other event, say A, is between zero and one, i.e., 0 ≤ P(A) ≤ 1 for any event A.

Set notation and set algebra, such as $\cup$, $\cap$, $\in$, and complement ($A' = A^c = \bar{A}$) are used in defining some events.

Activity 1: Consider a single roll of two dice, a red one and a green one. The table below shows the set of outcomes in the sample space, S. Each outcome is a pair of numbers--the number appearing on the red die and the number appearing on the green die. The event that consists of the whole sample space is the event that some one of the outcomes occurs. This event is certain to happen; if we roll the dice, the outcome cannot be something other than one of the 36 outcomes listed in the table. Therefore, the probability associated with the event S is P(S) = 1.

	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
Number on Green Die	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)
		1	2	3	4	5	6

Number on Red Die

If the dice are fair, then each of the 36 possible outcomes is equally likely. Also only one of the outcomes can happen on any roll of the dice. Consequently, to find the probability of the event consisting of just one of the outcomes (any one), we simply divide the probability of the entire sample space by 36.

Therefore, for example:

P(1,1) = $\frac{1}{36}$

P(4,5) = $\frac{1}{36}$

P(2,3) = $\frac{1}{36}$

Any event is a subset of the entire sample space. SO, the sample space of the given event is less than or equal to the entire sample space. For each event, the probability will be calculated in two different ways.