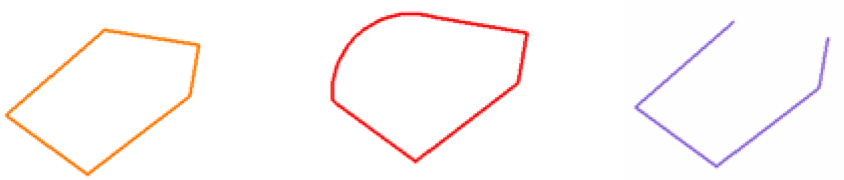
Polygons

We are familiar with triangles and we studied them in details in the previous chapter. But in fact triangles are special kinds of polygons. In this chapter we will focus on polygons and we will study in details the four sided polygons. But what are polygons?

Definition 1: A polygon is a 2-dimensional shape. It is a closed plane figure made up of straight lines. The sides do not cross each other. Exactly two sides meet at every vertex.

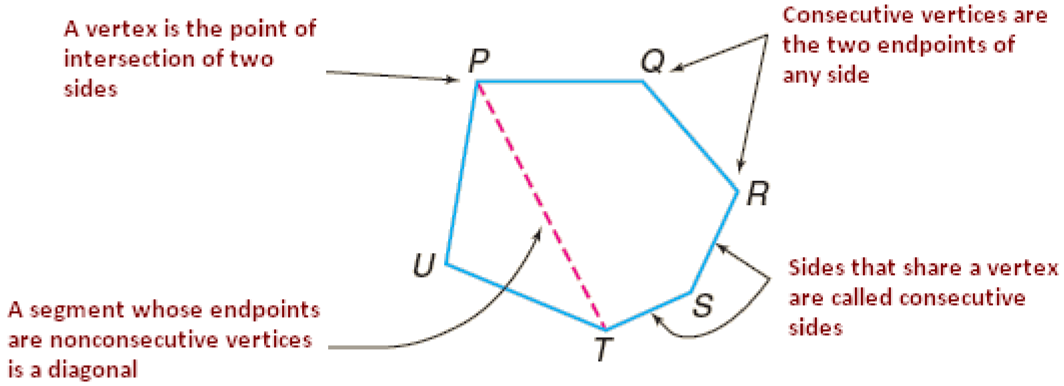


Polygon (straight sides) — **Not** a Polygon (has a curve) — **Not** a Polygon (open, not closed)

A polygon is named by the number of its sides or angles. A triangle is a polygon with three sides. The prefix *tri-* means three. Prefixes are also used to name other polygons. A polygon with n sides is called n-gon.

# of Sides	3	4	5	6	7	8
Name	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon

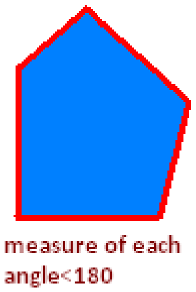
Check the Hexagon below to understand the different parts of the polygons



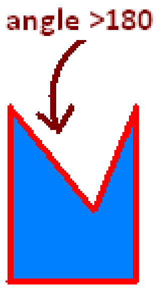
To name a polygon we start with one vertex and we move in a circular way around the polygon to list the letters in order. For instance, the above hexagon is called PQRSTU or SRQPUT or UPQRST…etc.

Definition 2: A polygon is convex if no line that contains a side of the polygon contains a point in the interior of the polygon. A convex polygon has no angles pointing inwards. More precisely, no internal angles can be more than 180°.

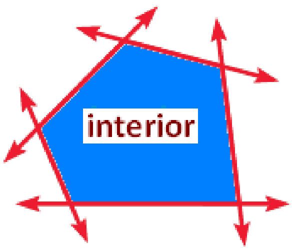
Definition 3: A polygon that is not convex is called concave. If there are any internal angles greater than 180° then it is concave.



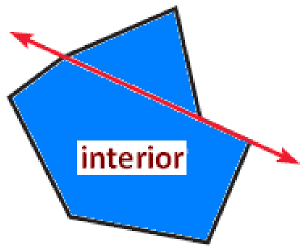
Convex



Concave



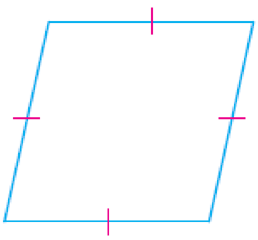
Convex

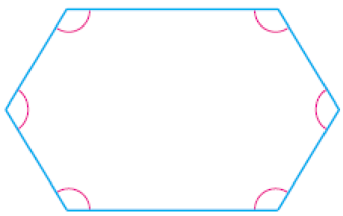


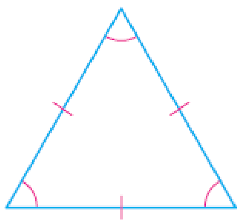
Concave

What if the measures of the sides of the polygons are equal? What if the measures of the angles of the polygons are equal? For such cases we will define the equilateral, equiangular and regular polygons

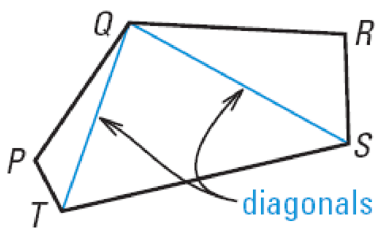
Definition 4: An equilateral polygon has all sides congruent, and an equiangular polygon has all angles congruent. A regular polygon is both equilateral and equiangular.


Equilateral but not equiangular

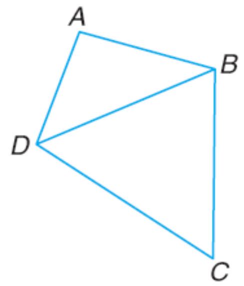

Equiangular, but not equilateral


Regular, both equilateral and equiangular

Definition 5: A diagonal of a polygon is a segment that joins two *nonconsecutive* vertices. Polygon *PQRST* has 2 diagonals from point Q, $\overline{QT}$ and $\overline{QS}$



Activity 1: Find the sum of measures of interior angles of a quadrilateral.
Step 1: Draw a convex quadrilateral.
Step 2: Choose one vertex and draw all possible diagonals from that vertex.
Step 3: How many triangles are formed?
Step4: What is the sum of measures of the interior angles of a quadrilateral?



From the above activity we deduce that the quadrilateral is made up of two triangles and therefore, the sum of the measures of the interior angles is equal to $2(sum\ of\ \ angles\ of\ \ triangle) = 2(180^0) = 360^0$

This activity can be repeated with a pentagon, a hexagon, a heptagon....You extended this pattern to other convex polygons and found the sum of interior angles of a polygon with *n* sides. The results are stated in the following theorem.

Theorem 1: If a convex polygon has n sides, then the sum of the measures of its interior angles is $(n-2)180^0$.

Let us check the table below that relates different facts about specific polygons

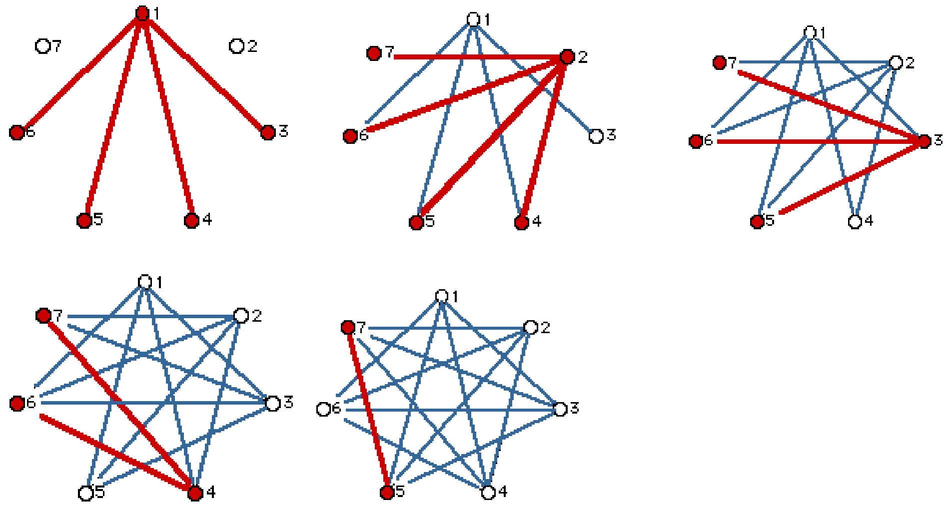
Convex Polygon	Number of sides	Number of diagonals from 1 vertex	Number of triangles	Sum of interior angles
Quadrilateral	4	1	2	$2(180^0) = 360^0$
Pentagon	5	2	3	$3(180^0) = 540^0$
Hexagon	6	3	4	$4(180^0) = 720^0$
Heptagon	7	4	5	$5(180^0) = 900^0$
Octagon	8	5	6	$6(180^0) = 1080^0$
n-gon	n	n-3	n-2	$(n-2)180^0$

All interior angles of a **regular polygon** have the same measure. So, to find the measure of one interior angle of a regular polygon we divide the sum of measures of interior angles by the number of interior angles.

Rule 1: The measure of one interior angle of a regular polygon is:

$$Measure\text{ of one interior angle}=\frac{\text{sum of measures of interior angles}}{\text{number of interior angles}}=\frac{(n-2)180^0}{n}$$

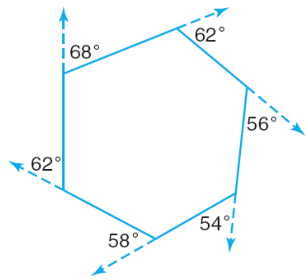
The total number of diagonals of a quadrilateral is 2. The total number of diagonals of a pentagon is 5. The total number of diagonals of a hexagon is 9. What about n-gon? Let us look at the diagonals of a 7-gon.



Rule 2: The total number of diagonals of a polygon is:

$$\frac{n(n-3)}{2}$$

<u>Exterior Angles of Polygons</u>



We can extend the sides of any convex polygon to form exterior angles. If you add the measures of the exterior angles in the hexagon, you find that the sum is 360^0.

The figure suggests a method for finding the sum of the measures of the exterior angles of a convex polygon. Let us derive this formula together.

Consider a polygon of n sides, when you extend *n* sides of a polygon, *n* linear pairs of angles are formed. The sum of the angle measures in each linear pair is 180.

$$\begin{aligned}\text{Sum of measures of ext. } \angle\text{s} &= \text{sum of measures of linear pairs-sum of measures of int. } \angle\text{s} \\ &= n \bullet 180^0 - 180(n-2)^0 \\ &= 180^0 n - 180^0 n + 360^0 \\ &= 360^0\end{aligned}$$

So, we conclude that the sum of the exterior angle measures is 360^0 for any convex polygon. Therefore the sum of the exterior angles of a polygon is independent of the number of sides.

Theorem 2: In **any convex polygon**, the sum of the measures of the exterior angles, one at each vertex, is 360^0.

Rule 3: The measure of one exterior angle of a regular polygon is:

$$\frac{360^0}{\textit{number} \text{ of sides}} = \frac{360^0}{n}$$